Musical Languages and Formal Sciences

ICE Department seminar, June 2022

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Institut Mines-Télécom

Prolegomenon

Johann Sebastian Bach, Anonym, 1725 (British Museum)

Prolegomenon

$$\zeta(s) = \sum_{n=1}^{+\infty} \frac{1}{n^s}$$

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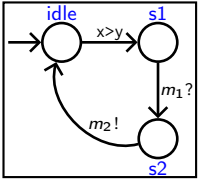

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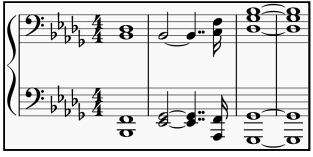

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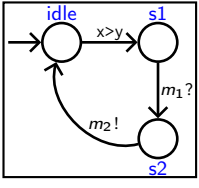

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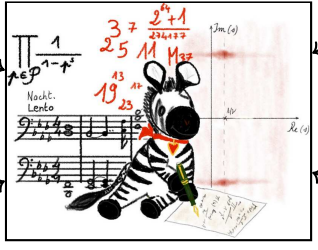

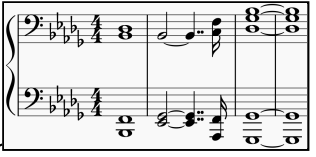

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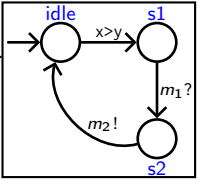
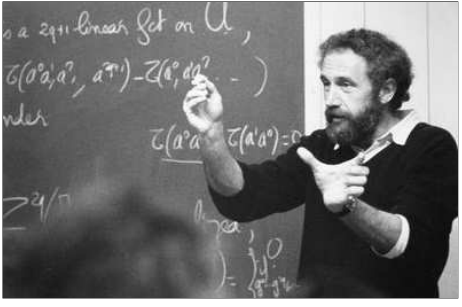


Prolegomenon

"Il m'est arrivé d'avoir un apport de l'extérieur par une oeuvre musicale pour un problème que je me posais, et que cet apport musical soit plus important que si j'avais lu un texte mathématique. Il m'est arrivé d'écouter des oeuvres musicales [...] qui avaient un sens qui cadrait avec une espèce d'intuition que j'avais à un moment donné, mais que je ne pouvais pas traduire autrement, que je ne pouvais pas traduire par des mots : mais il y avait, par exemple, un prélude qui correspondait exactement à cette intuition. Et je ne savais pas pourquoi. Donc là, il y a quelque chose, à mon avis."

Alain Connes, June 2011 [3]



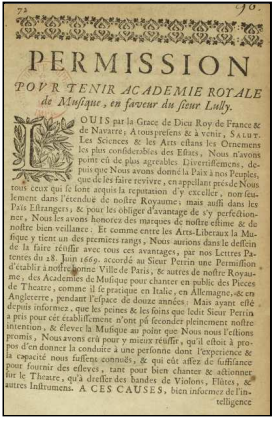
Alain Connes, photographed by Dirk Ferraus, 1989 (Oberwolfach Photo Collection)

Plan

A (very brief) history of musical interpretation [1]

The Beggars Opera, Anonym, 1728 (MET)

A (very brief) history of musical interpretation [1]



PERMISSION

POVR TENIR ACADEMIE ROYALE de Musique, en faveur du sieur Lully.

LOUIS par la Grace de Dieu Roy de France & de Navarre; A tous presens & à venir, SALUT. Les Sciences & les Arts eſtans les Ornemens les plus conſiderables des Eſtats, Nous n'avons point eû de plus agreables Divertiſſemens, depuis que Nous avons donné la Paix à nos Peuples, que de les faire revivre, en appellant prés de Nous tous ceux qui ſe ſont acquis la reputation d'y exceller, non ſeulement dans l'etenduë de noſtre Royaume; mais auſſi dans les Païs Eſtrangers; & pour les obliger d'avantage de s'y perfectionner, Nous les avons honorez des marques de noſtre eſtime & de noſtre bien veillance. Et comme entre les Arts-Liberaux la Muſique y tient un des premiers rangs, Nous aurions dans le deſſein de la faire réuſſir avec tous ces avantages, par nos Lettres Patentes du 28. Juin 1669. accordé au Sieur Perrin une Permiſſion d'eſtablir à noſtre bonne Ville de Paris, & autres de noſtre Royaume, des Academies de Muſique pour chanter en public des Pieces de Theatre, comme il ſe pratique en Italie, en Allemagne, & en Angleterre, pendant l'eſpace de douze années: Mais ayant eſté depuis informez, que les peines & les ſoins que ledit Sieur Perrin a pris pour cét établiſſement n'ont pû feconder pleinement noſtre intention, & élever la Muſique au point que Nous nous l'eſtions promis, Nous avons crû pour y mieux réuſſir, qu'il eſtoit à propos d'en donner la conduite à une perſonne dont l'experience & la capacité nous fuſſent connuës, & qui eût aſſez de ſuffiſance pour fournir des eſleves, tant pour bien chanter & actionner ſur le Theatre, qu'à dreſſer des bandes de Violons, Flûtes, & autres Inſtrumens. A CES CAUSES, bien informez de l'intelligence

Lettres patentes portant création de l'Académie Royale de Musique, 1669 (BNF)

Jean-Baptiste Lully, Paul Mignard, ca. 1650 (Musée Condé)

A (very brief) history of musical interpretation [2]

Maurice Ravel, 1925 (BNF)

Igor Stravinsky, photographed by Robert Regassi, 1921

A (very brief) history of musical interpretation

"L'interprète est en réalité un exécutant, l'exécutant direct de la volonté du compositeur. Il n'apporte rien qui ne soit déjà dans l'œuvre. S'il a du talent, il laisse entrevoir la vérité de l'œuvre qui seule est géniale et se reflète en lui. Il ne doit pas dominer la musique mais se dissoudre en elle. [...] J'ai toujours été sûr, pour chaque œuvre, que c'était ainsi et pas autrement qu'il fallait la jouer. Et pourquoi ? C'est très simple ; parce que je regardais attentivement la partition."

Sviatoslav Richter [4]

Sviatoslav Richter
(www.brunomonsaingeon.com)

A (very brief) history of musical interpretation

Pierre Boulez : “La chose écrite [...] ne transmet qu’une part de l’invention” [2]

Historically informed performance

- Period instruments
- Period tuning-fork (e.g., 415 Hz)
- Adjusted musical ensembles
- Interpretation style (e.g., ornaments)
- etc.

Jordi Savall, photographed by Yannick Coupannec, 2016 (France Culture)

Plan

The written musical language

Two semantic levels

Musical language $\neq$ written musical language

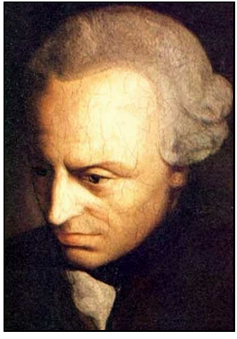
The main musical parameters

- Pitch
- Duration
- Timbre
- Volume
- Articulation
- Spatialisation

The written musical language

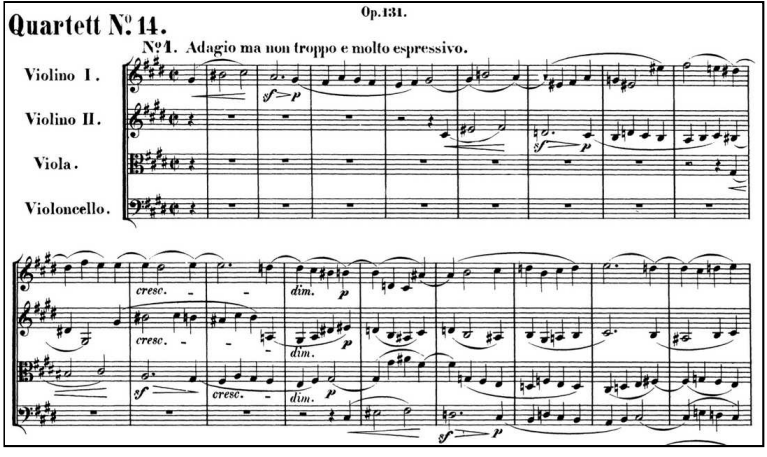
Critic of pure written musical language

- Formal elements
- Non-formal elements



Immanuel Kant, Anonym, ca. 1790

The written musical language



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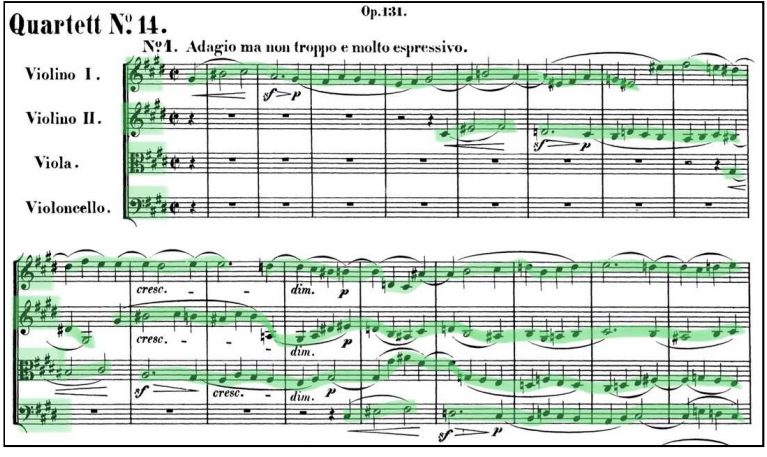

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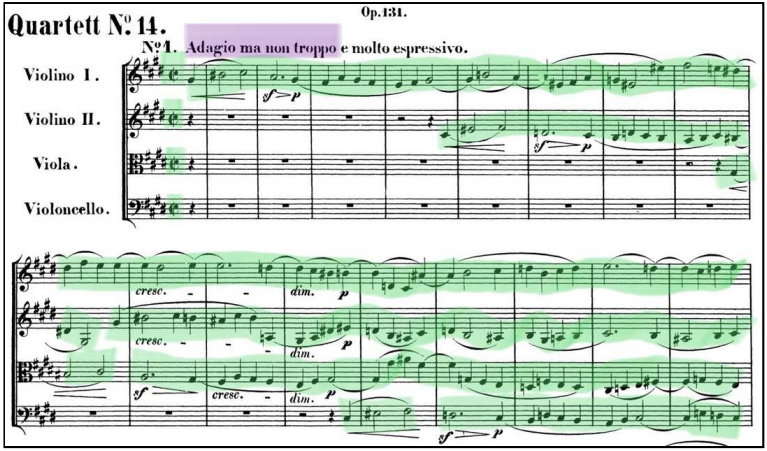

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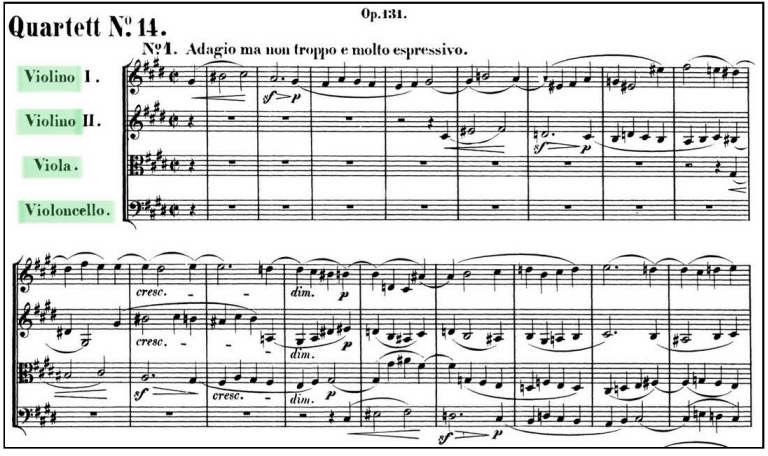

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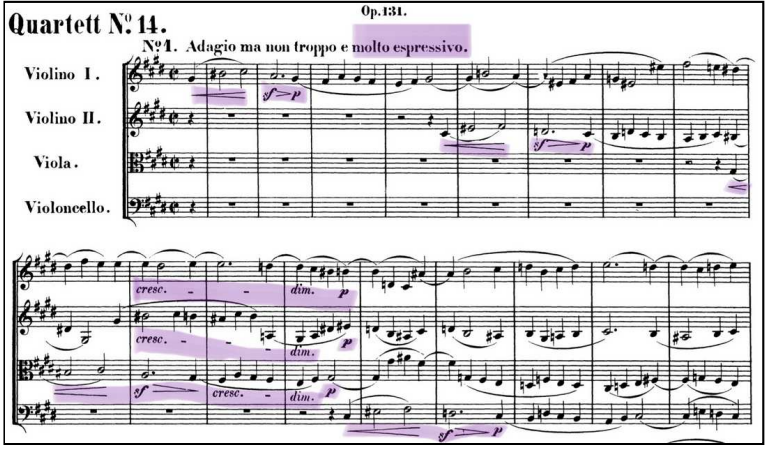

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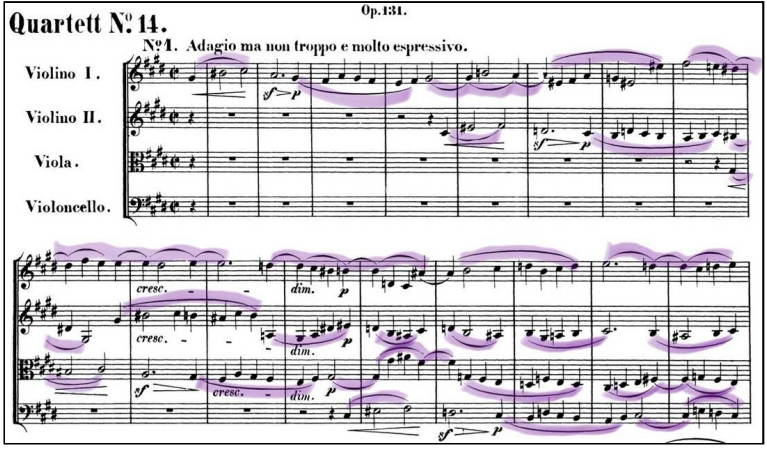

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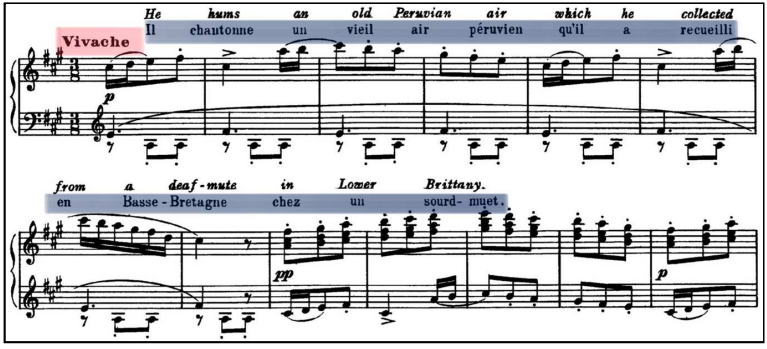
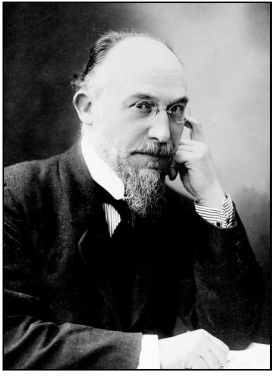




Érik Satie, photographed by Henri Manuel, ca. 1920

The written musical language: summary

Formally defined parameters

- Pitch
- Timbre
- Duration...

The written musical language: summary

Formally defined parameters

- Pitch
- Timbre
- Duration...

Non-formally defined parameters

- ... Duration !
- Volume
- Articulation

The written musical language: summary

Pitch

- Intervals
  - Vertical component (harmony)
  - Horizontal component (melody)
  - Fundamental elements of the *modes*

The written musical language: summary

Pitch

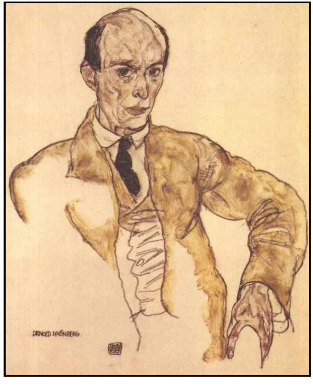
- Intervals
  - Vertical component (harmony)
  - Horizontal component (melody)
  - Fundamental elements of the *modes*
- Musical language and modes
  - Tonal language (major scale, minor scale)
  - Modal language (Dorian mode, Messiaen's seven modes of limited transposition, etc.)
  - Atonal language

Plan

Plan

Serial music

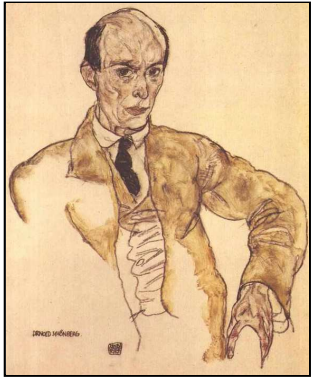
- Music



Arnold Schoenberg, Egon Schiele, 1917

Serial music

- Music
- **Seri**al



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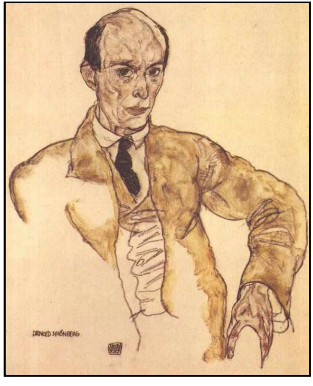
Serial music

- Music
- **Seri**al
- Dodecaphonic



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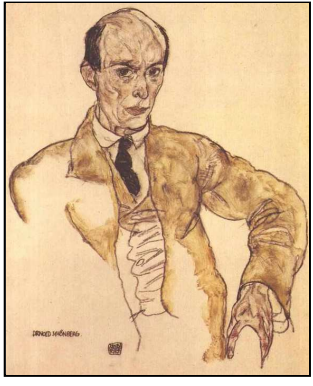
Serial music

- Music
- **Seri**al
- Dodecaphonic

Dodecaphonic serie

- Western music relies on twelve tones
- C, C♯ =D♭, D, D♯ =Mi♭, E, F, F♯ =G♭, G, G♯ =A♭, A, A♯ =B♭, B



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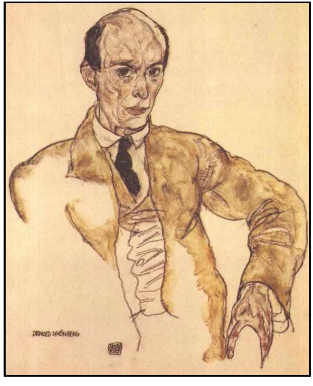
Serial music

Core material: the serie

- Set of pitches $\mathbb{H}$

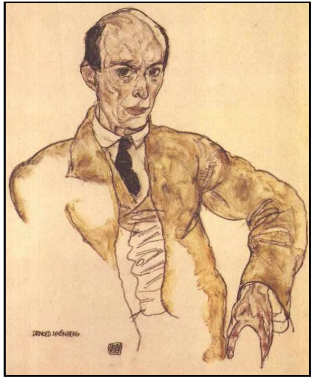


Arnold Schoenberg, Egon Schiele, 1917

Serial music

Core material: the serie

- Set of pitches $\mathbb{H}$
- Dodecaphonic scale $\mathbb{G} = \{C, C\sharp, B, B\sharp, E, F, F\sharp, G, G\sharp, A, A\sharp, B\}$



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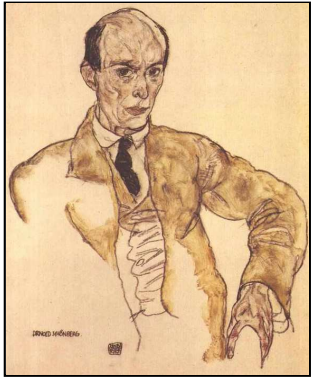
- $h : \mathbb{H} \twoheadrightarrow \mathbb{G}$



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Serial music

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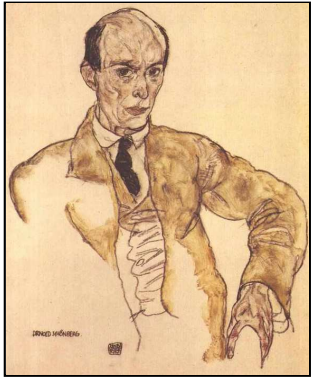
- $h : \mathbb{H} \twoheadrightarrow \mathbb{G}$
- $h_s : \mathbb{H}^p \twoheadrightarrow \mathbb{G}^p$
  $\langle n_1, ..., n_p \rangle \mapsto \langle h(n_1), ..., h(n_p) \rangle$



Arnold Schoenberg, Egon Schiele, 1917

Serial music

Core material: the serie

- Set of pitches $\mathbb{H}$
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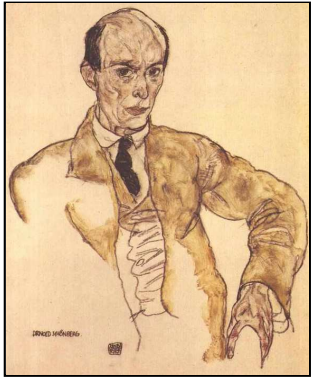
- $h : \mathbb{H} \twoheadrightarrow \mathbb{G}$
- $h_s : \mathbb{H}^p \twoheadrightarrow \mathbb{G}^p$
  $\langle n_1, ..., n_p \rangle \mapsto \langle h(n_1), ..., h(n_p) \rangle$
- $\mathcal{S} \in \mathbb{H}^{12}$ such that
  $h_s(\mathcal{S}) \in \{\sigma(\mathbb{G}) \mid \sigma \in \mathfrak{S}(\mathbb{G})\}$



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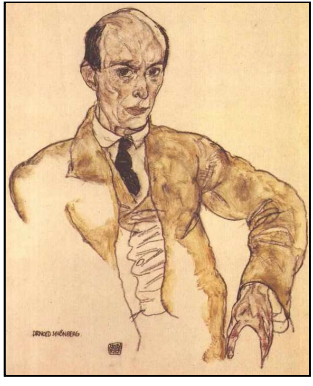
Serial music

And now, serial transformations!

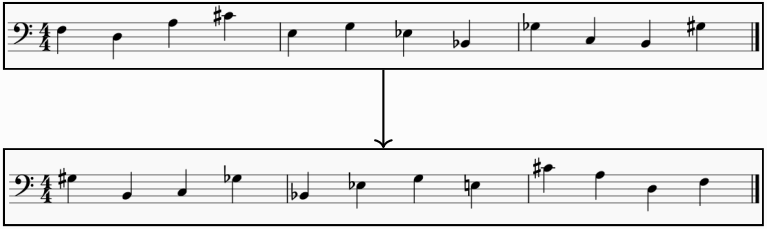
- $t(\mathcal{S})$
- retrograde
- inversion
- retrograde inversion
- transposition
- 48 distinct series



Arnold Schoenberg, Egon Schiele, 1917

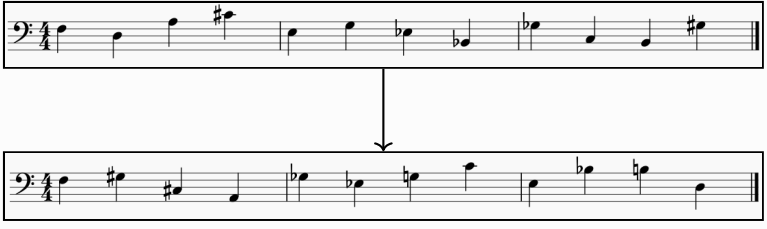
Serial transformations (1/4)

Retrograde



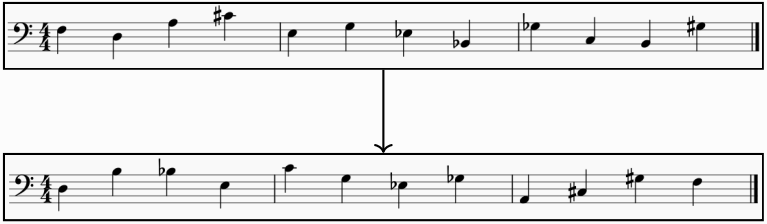
Serial transformations (2/4)

Intervallic inversion



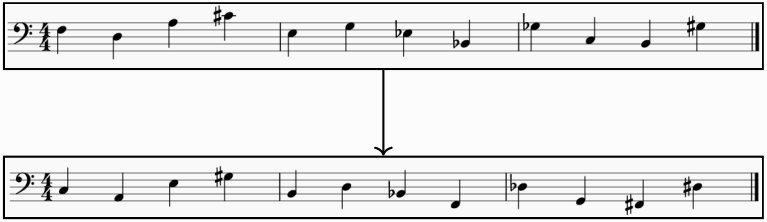
Serial transformations (3/4)

Retrograde inversion



Serial transformations (4/4)

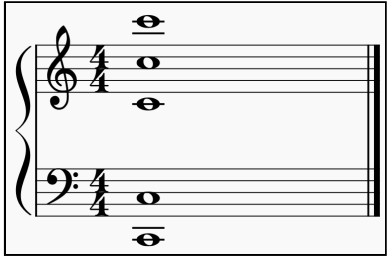
Transposition



Pitch-Class Set Theory [5]

Pitch-Class

The pitch-class of an element $e \in \mathbb{G}$ is the set of the notes whose pitch x is such that $h(x) = e$.



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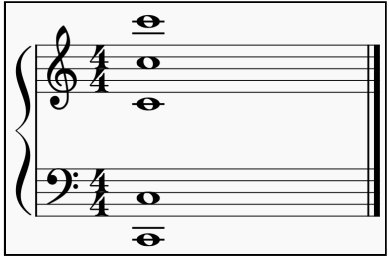
The pitch-class of an element $e \in \mathbb{G}$ is the set of the notes whose pitch x is such that $h(x) = e$.

Example

$\overline{C} = \{\ldots, C_2, C_3, C_4, C_5, C_6, \ldots\}$



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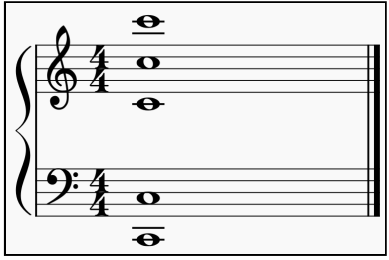
Example

$\overline{C} = \{\ldots, C_2, C_3, C_4, C_5, C_6, \ldots\}$

Does that ring a bell?

$ECH \approx \mathbb{Z}/12\mathbb{Z}$



Allen Forte Equivalence

- $\overline{C} \mapsto 0$, $\overline{C\sharp} \mapsto 1$, $\overline{D} \mapsto 2$, $\overline{D\sharp} \mapsto 3$, $\overline{E} \mapsto 4$, $\overline{F} \mapsto 5$, $\overline{F\sharp} \mapsto 6$, $\overline{G} \mapsto 7$, $\overline{G\sharp} \mapsto 8$, $\overline{A} \mapsto 9$, $\overline{A\sharp} \mapsto 10$, $\overline{B} \mapsto 11$

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- Each note is depicted by the integer coding its pitch-class

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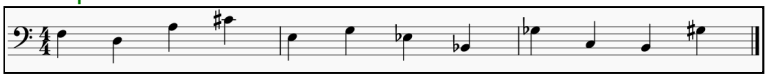
- Each note is depicted by the integer coding its pitch-class

Example



$\langle 5, 2, 9, 1, 4, 7, 3, 10, 6, 0, 11, 8 \rangle$

Pitch-Class Set Theory [5]

Then, we study the pitch-class "sets"

- Actually, sets are often $n-$uples

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- But also derive other objects, such as *intervallic vectors*
  - These vectors are useful for analyzing the intervallic structure of a pitch-class set

Pitch-Class Set Theory [5] and serial transformations

Let $\mathcal{S}$ be a serie. We denote with $\mathcal{S}_h$ its pitch-class 12-uple. We can easily define the transformations of $\mathcal{S}_h$ related to the serial transformations of $\mathcal{S}$.

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Plan

Compose with a dinosaur
Methodology

TTool, Ludovic Apvrille

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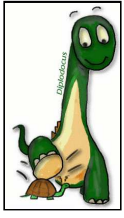

Piano Fazioli (www.pianoshanlet.fr)

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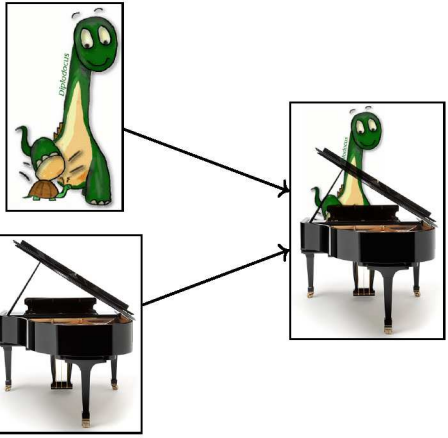

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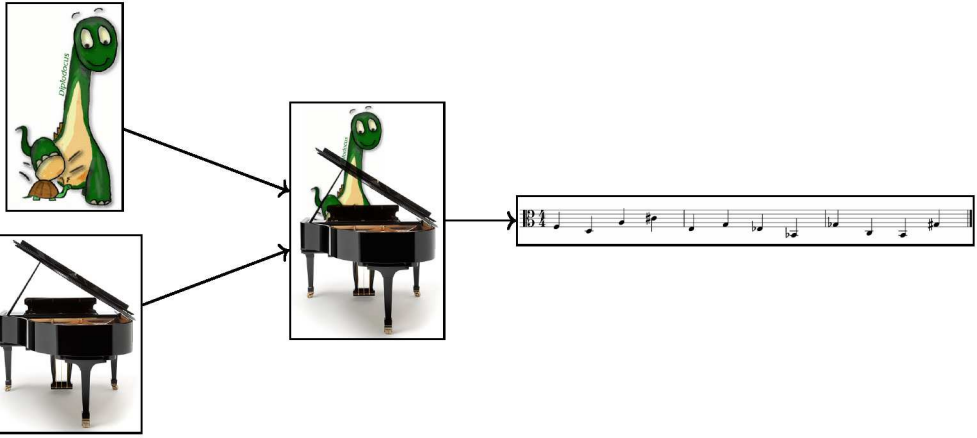

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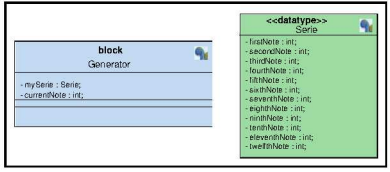

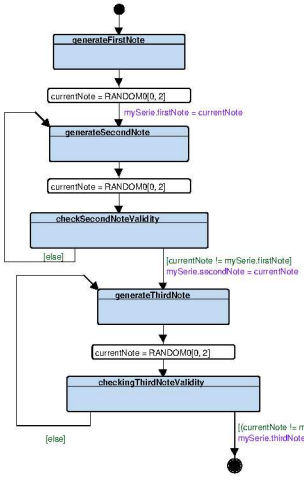

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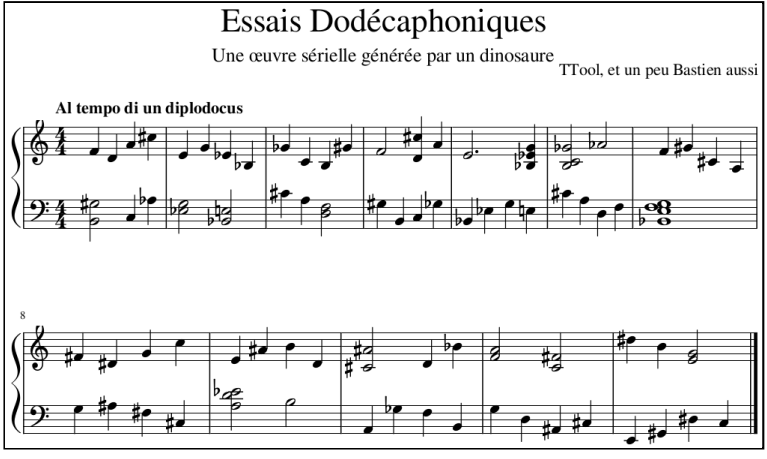

Plan

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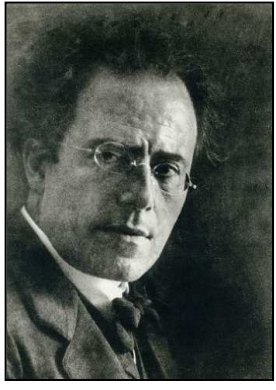
- What about silence ?
- Evolution of musical languages

Modern music and modern mathematics

- 1908 : Zermelo set theory
- 1909-1910 : Mahler's 9th Symphony
- 1921 : Schoenberg's first serial work
- 1922 : ZF
- 1935 : Bourbaki
- 1949 : Darmstadt Summer Course, Messiaen creates *Mode de valeurs et d'intensité*



Gustav Mahler, photographed by Friedrich Victor Spitzer, 1905

Plan

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