



Musical Languages and Formal Sciences

ICE Department seminar, June 2022

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June 2, 2022



Prolegomenon



Johann Sebastian Bach,
Anonym, 1725 (British
Museum)

Prolegomenon

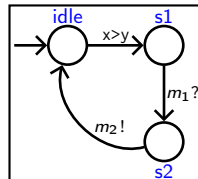


$$\zeta(s) = \sum_{n=1}^{+\infty} \frac{1}{n^s}$$

Prolegomenon



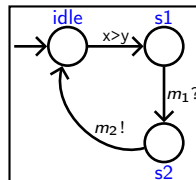
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Prolegomenon



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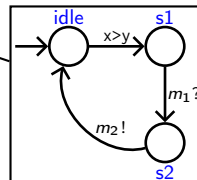
Prolegomenon



Handwritten musical notation and mathematical annotations:

- Time signature: $\frac{1}{4}$
- Tempo: *Nacht. Lento*
- Key signature: G major
- Handwritten numbers: $3, 7, 2^4, 1, 2, 5, 11, 13, 17, 19, 23$
- Handwritten formula: $\frac{2^4 + 1}{2^3 + 1}$
- Handwritten text: M_{37}
- Handwritten text: $\text{Im}(s)$
- Handwritten text: $\text{Re}(s)$
- A zebra character is sitting on a piece of paper with musical notation.

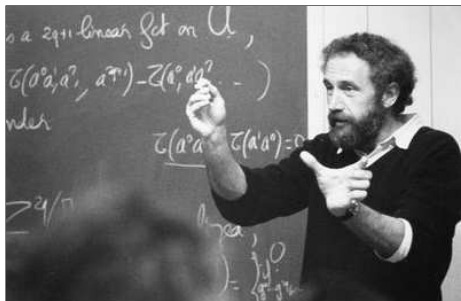
$$\zeta(s) = \sum_{n=1}^{+\infty} \frac{1}{n^s}$$



Prolegomenon

“Il m’est arrivé d’avoir un apport de l’extérieur par une oeuvre musicale pour un problème que je me posais, et que cet apport musical soit plus important que si j’avais lu un texte mathématique. Il m’est arrivé d’écouter des oeuvres musicales [...] qui avaient un sens qui cadrerait avec une espèce d’intuition que j’avais à un moment donné, mais que je ne pouvais pas traduire autrement, que je ne pouvais pas traduire par des mots : mais il y avait, par exemple, un prélude qui correspondait exactement à cette intuition. Et je ne savais pas pourquoi. Donc là, il y a quelque chose, à mon avis.”

Alain Connes, June 2011 [3]



Alain Connes, photographed by Dirk Ferraus, 1989 (Oberwolfach Photo Collection)

Plan

Musical interpretation and fidelity to the score

The musical language: a formal language?

A set-theoretic approach

Serial music and *Pitch-Class Set Theory*

Compose with a dinosaur

And now?

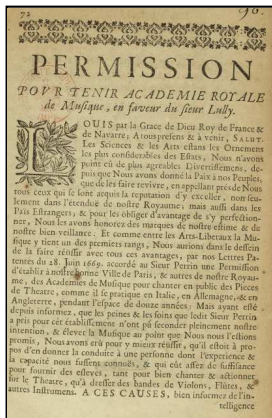
Bibliographie

A (very brief) history of musical interpretation [1]



The Beggars Opera, Anonym, 1728 (MET)

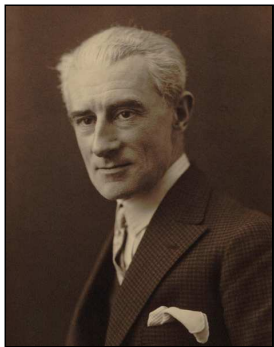
A (very brief) history of musical interpretation [1]



Jean-Baptiste Lully, Paul Mignard,
ca. 1650 (Musée Condé)

*Lettres patentes portant création de
l'Académie Royale de Musique, 1669 (BNF)*

A (very brief) history of musical interpretation [2]



Maurice Ravel, 1925 (BNF)



Igor Stravinsky, photographed by
Robert Regassi, 1921

A (very brief) history of musical interpretation

“L’interprète est en réalité un exécutant, l’exécutant direct de la volonté du compositeur. Il n’apporte rien qui ne soit déjà dans l’œuvre. S’il a du talent, il laisse entrevoir la vérité de l’œuvre qui seule est géniale et se reflète en lui. Il ne doit pas dominer la musique mais se dissoudre en elle. [...] J’ai toujours été sûr, pour chaque œuvre, que c’était ainsi et pas autrement qu’il fallait la jouer. Et pourquoi ? C’est très simple ; parce que je regardais attentivement la partition.”

Sviatoslav Richter [4]



Sviatoslav Richter
(www.brunomonsaingeon.com)

A (very brief) history of musical interpretation

Pierre Boulez : “La chose écrite [...] ne transmet qu’une part de l’invention” [2]

Historically informed performance

- ▶ Period instruments
- ▶ Period tuning-fork (e.g., 415 Hz)
- ▶ Adjusted musical ensembles
- ▶ Interpretation style (e.g., ornaments)
- ▶ etc.



Jordi Savall, photographed by Yannick Coupanec, 2016 (France Culture)

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The written musical language

Two semantic levels

Musical language \neq written musical language

The main musical parameters

- ▶ Pitch
- ▶ Duration
- ▶ Timbre
- ▶ Volume
- ▶ Articulation
- ▶ Spatialisation

The written musical language

Critic of pure written musical language

- ▶ Formal elements
- ▶ Non-formal elements



Immanuel Kant, Anonym, ca. 1790

The written musical language

Quartett N° 14. Op.131.
N°1. Adagio ma non troppo e molto espressivo.

Violino I.
Violino II.
Viola.
Violoncello.

The image displays a page of a musical score for a string quartet. The title is "Quartett N° 14." with the opus number "Op.131." and the movement "N°1. Adagio ma non troppo e molto espressivo." The score is arranged in two systems. The first system shows the beginning of the piece with dynamic markings *sf* and *p*. The second system continues the piece with dynamic markings *cresc.*, *dim.*, and *p*. The instruments are Violino I, Violino II, Viola, and Violoncello. The key signature is G major (one sharp) and the time signature is 3/4.

The written musical language

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The written musical language

Quartett N° 14. Op.131.
N°1. Adagio ma non troppo e molto espressivo.

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The image displays a musical score for a string quartet. The title is "Quartett N° 14." Op. 131, N°1. The tempo and expression are "Adagio ma non troppo e molto espressivo." The score is for Violino I, Violino II, Viola, and Violoncello. The music is in G major and 3/4 time. The first system shows the beginning of the piece with dynamics *sf* and *p*. The second system shows a crescendo and decrescendo section with dynamics *cresc.*, *dim.*, and *p*. The score is highlighted with green shading.

The written musical language

Quartett N° 14. Op.131.

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Viola.

Violoncello.

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The written musical language

Quartett N° 14. Op. 131.

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N°1. Adagio ma non troppo e molto espressivo.

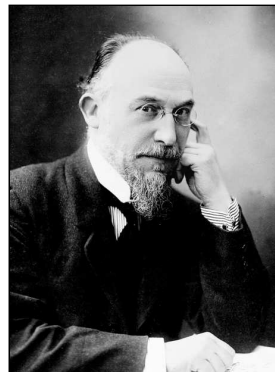
Violino I.
Violino II.
Viola.
Violoncello.

The written musical language

Vivache

He hums an old Peruvian air which he collected
Il chantonne un vieil air péruvien qu'il a recueilli

from a deaf-mute in Lower Brittany.
en Basse-Bretagne chez un sourd-muet.

The image shows a musical score for Erik Satie's piece 'Vivache'. It consists of two systems of music. The first system features a vocal line in treble clef with lyrics in English and French, and a piano accompaniment in bass clef. The second system continues the piano accompaniment. The score includes dynamic markings such as 'p' and 'pp', and a tempo marking 'Vivache'. The key signature is one sharp (F#).

Érik Satie, photographed by Henri Manuel, ca. 1920

The written musical language: summary

Formally defined parameters

- ▶ Pitch
- ▶ Timbre
- ▶ Duration...

The written musical language: summary

Formally defined parameters

- ▶ Pitch
- ▶ Timbre
- ▶ Duration...

Non-formally defined parameters

- ▶ ... Duration !
- ▶ Volume
- ▶ Articulation

The written musical language: summary

Pitch

- ▶ Intervals
 - ▶ Vertical component (harmony)
 - ▶ Horizontal component (melody)
 - ▶ Fundamental elements of the *modes*

The written musical language: summary

Pitch

- ▶ Intervals
 - ▶ Vertical component (harmony)
 - ▶ Horizontal component (melody)
 - ▶ Fundamental elements of the *modes*
- ▶ Musical language and modes
 - ▶ Tonal language (major scale, minor scale)
 - ▶ Modal language (Dorian mode, Messiaen's seven modes of limited transposition, etc.)
 - ▶ Atonal language

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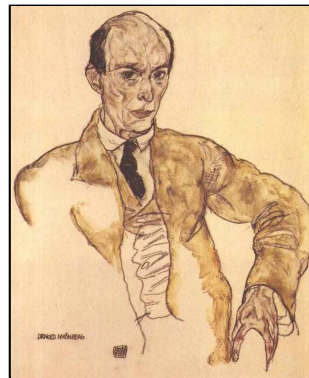
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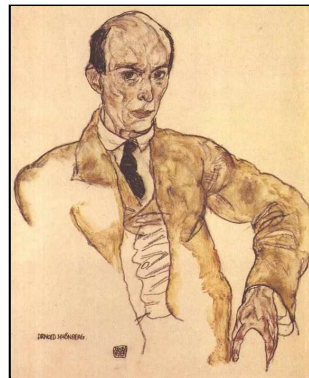
► Music



Arnold Schoenberg, Egon Schiele, 1917

Serial music

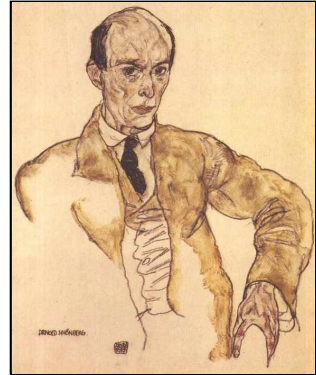
- ▶ Music
- ▶ Serial



Arnold Schoenberg, Egon Schiele, 1917

Serial music

- ▶ Music
- ▶ Serial
- ▶ Dodecaphonic



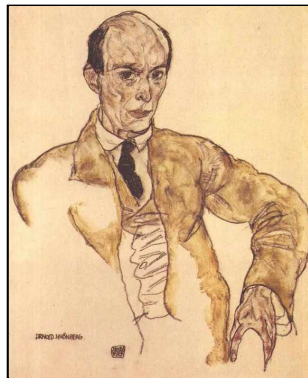
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Serial music

- ▶ Music
- ▶ Serial
- ▶ Dodecaphonic

Dodecaphonic serie

- ▶ Western music relies on twelve tones
- ▶ C, C \sharp = D \flat , D, D \sharp = M \flat , E, F, F \sharp = G \flat , G, G \sharp = A \flat , A, A \sharp = B \flat , B

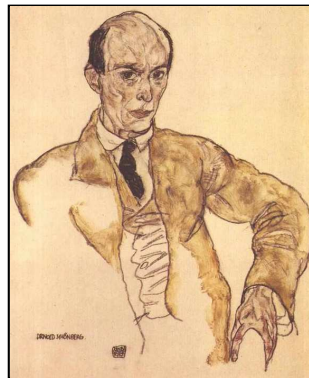


Arnold Schoenberg, Egon Schiele, 1917

Serial music

Core material: the serie

- ▶ Set of pitches III

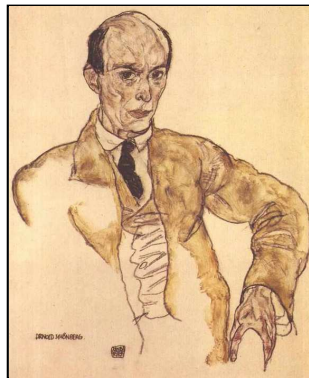


Arnold Schoenberg, Egon Schiele, 1917

Serial music

Core material: the serie

- ▶ Set of pitches III
- ▶ Dodecaphonic scale $\mathbb{G} = \{C, C\#, B, B\#, E, F, F\#, G, G\#, A, A\#, B\}$

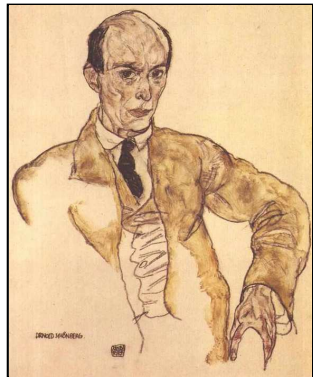


Arnold Schoenberg, Egon Schiele, 1917

Serial music

Core material: the serie

- ▶ Set of pitches \mathbb{H}
- ▶ Dodecaptonic scale $\mathbb{G} = \{C, C\#, B, B\#, E, F, F\#, G, G\#, A, A\#, B\}$
- ▶ $h : \mathbb{H} \rightarrow \mathbb{G}$

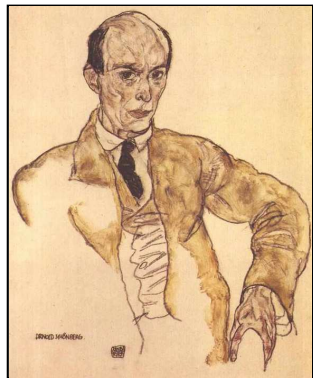


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- ▶ $h : \mathbb{H} \rightarrow \mathbb{G}$
- ▶ $h_s : \mathbb{H}^p \rightarrow \mathbb{G}^p$
 $\langle n_1, \dots, n_p \rangle \mapsto \langle h(n_1), \dots, h(n_p) \rangle$

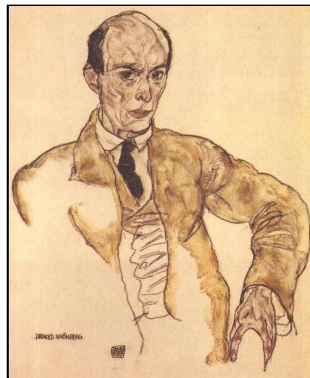


Arnold Schoenberg, Egon Schiele, 1917

Serial music

Core material: the serie

- ▶ Set of pitches \mathbb{H}
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- ▶ $h : \mathbb{H} \rightarrow \mathbb{G}$
- ▶ $h_s : \mathbb{H}^p \rightarrow \mathbb{G}^p$
 $\langle n_1, \dots, n_p \rangle \mapsto \langle h(n_1), \dots, h(n_p) \rangle$
- ▶ $S \in \mathbb{H}^{12}$ such that
 $h_s(S) \in \{\sigma(\mathbb{G}) \mid \sigma \in \mathfrak{S}(\mathbb{G})\}$

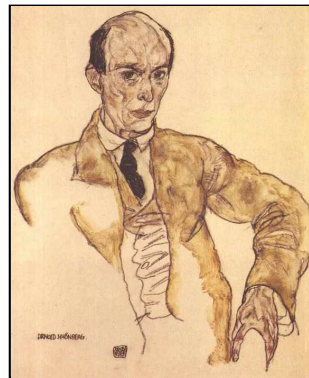


Arnold Schoenberg, Egon Schiele, 1917

Serial music

And now, serial transformations!

- ▶ $t(S)$
- ▶ retrograde
- ▶ inversion
- ▶ retrograde inversion
- ▶ transposition
- ▶ 48 distinct series



Arnold Schoenberg, Egon Schiele, 1917

Serial transformations (1/4)

Retrograde

The image displays two musical staves in bass clef, 4/4 time. The top staff contains the following notes: G₂, A₂, B₂, C[#]₃, B₂, A₂, G₂, F₂, E₂, D₂, C₂, B₁. The bottom staff contains the notes: B₁, C₂, D₂, E₂, F₂, G₂, A₂, B₂, C[#]₃, B₂, A₂, G₂. A vertical arrow points from the top staff to the bottom staff, indicating a retrograde transformation.

Serial transformations (2/4)

Intervallic inversion

The image displays two musical staves in bass clef, 4/4 time, illustrating intervallic inversion. The top staff shows the original sequence of notes: C2, D2, E2, F#2, G2, A2, Bb2, C3, D3, E3, F#3, G3. The bottom staff shows the inverted sequence: C2, Bb1, Ab1, G1, F1, E1, D1, C1, Bb1, Ab1, G1, F1. A vertical arrow points from the first measure of the top staff to the first measure of the bottom staff, indicating the transformation process.

Serial transformations (3/4)

Retrograde inversion

The image displays two musical staves in bass clef with a 4/4 time signature. The top staff shows a sequence of notes: G2, A2, B2, C3, D3, E3, F3, G3. The bottom staff shows the retrograde inversion of this sequence: G3, F3, E3, D3, C3, B2, A2, G2. A vertical arrow points from the top staff to the bottom staff, indicating the transformation.

Serial transformations (4/4)

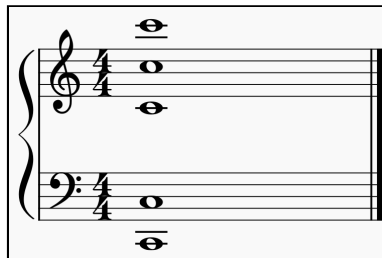
Transposition

The image displays two musical staves in bass clef with a 4/4 time signature. The top staff shows a melody starting on G2, moving to A2, B2, C#3, D3, E3, F#3, G3, A3, B3, C4, D4, E4, F#4, G4, A4, B4, C5. The bottom staff shows the same melody transposed down one octave, starting on G1 and ending on C4. A vertical arrow points from the top staff to the bottom staff, indicating the transposition.

Pitch-Class Set Theory [5]

Pitch-Class

The pitch-class of an element $e \in \mathbb{G}$ is the set of the notes whose pitch x is such that $h(x) = e$.



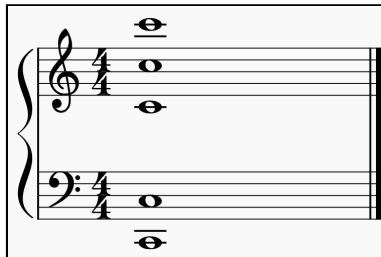
Pitch-Class Set Theory [5]

Pitch-Class

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Example

$$\overline{C} = \{\dots, C_2, C_3, C_4, C_5, C_6, \dots\}$$



Pitch-Class Set Theory [5]

Pitch-Class

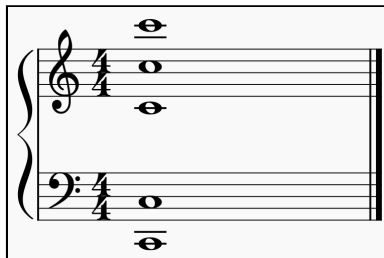
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$$\bar{C} = \{\dots, C_2, C_3, C_4, C_5, C_6, \dots\}$$

Does that ring a bell?

$$ECH \approx \mathbb{Z}/12\mathbb{Z}$$



Pitch-Class Set Theory [5]

Allen Forte Equivalence

- ▶ $\overline{C} \mapsto 0, \overline{C\sharp} \mapsto 1, \overline{D} \mapsto 2, \overline{D\sharp} \mapsto 3, \overline{E} \mapsto 4, \overline{F} \mapsto 5, \overline{F\sharp} \mapsto 6, \overline{G} \mapsto 7,$
 $\overline{G\sharp} \mapsto 8, \overline{A} \mapsto 9, \overline{A\sharp} \mapsto 10, \overline{B} \mapsto 11$

Pitch-Class Set Theory [5]

Allen Forte Equivalence

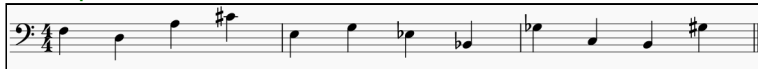
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 $\overline{G\sharp} \mapsto 8, \overline{A} \mapsto 9, \overline{A\sharp} \mapsto 10, \overline{B} \mapsto 11$
- ▶ Each note is depicted by the integer coding its pitch-class

Pitch-Class Set Theory [5]

Allen Forte Equivalence

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- ▶ Each note is depicted by the integer coding its pitch-class

Example



$\langle 5, 2, 9, 1, 4, 7, 3, 10, 6, 0, 11, 8 \rangle$

Pitch-Class Set Theory [5]

Then, we study the pitch-class “sets”

- ▶ Actually, sets are often n -uples

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- ▶ Pitch-class sets enable for modeling a set of notes, horizontally (melodic lines) or vertically (chords)

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- ▶ We can carry out transformations on these sets

Pitch-Class Set Theory [5]

Then, we study the pitch-class “sets”

- ▶ Actually, sets are often n -uples
- ▶ Pitch-class sets enable for modeling a set of notes, horizontally (melodic lines) or vertically (chords)
- ▶ We can carry out transformations on these sets
- ▶ But also derive other objects, such as *intervallic vectors*
 - ▶ These vectors are useful for analyzing the intervallic structure of a pitch-class set

Pitch-Class Set Theory [5] and serial transformations

Let \mathcal{S} be a serie. We denote with \mathcal{S}_h its pitch-class 12-uple. We can easily define the transformations of \mathcal{S}_h related to the serial transformations of \mathcal{S} .

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- ▶ n semi-tone transposition. $\mathcal{S}_{h_i} \xrightarrow{t} \mathcal{S}_{h_i} + n \bmod 12$

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- ▶ Intervallic inversion. $\mathcal{S}_{h_i} \xrightarrow{inv} 2 \times \mathcal{S}_{h_1} - \mathcal{S}_{h_i} \bmod 12$

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- ▶ Retrograde. $\mathcal{S}_{h_i} \xrightarrow{r} \mathcal{S}_{h_{13-i}}$

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- ▶ Retrograde. $\mathcal{S}_{h_i} \xrightarrow{r} \mathcal{S}_{h_{13-i}}$
- ▶ Retrograde inversion. $inv \circ r$



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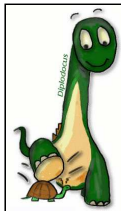
Compose with a dinosaur

And now?

Bibliographie

Compose with a dinosaur

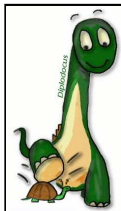
Methodology



TTool, Ludovic Aprville

Compose with a dinosaur

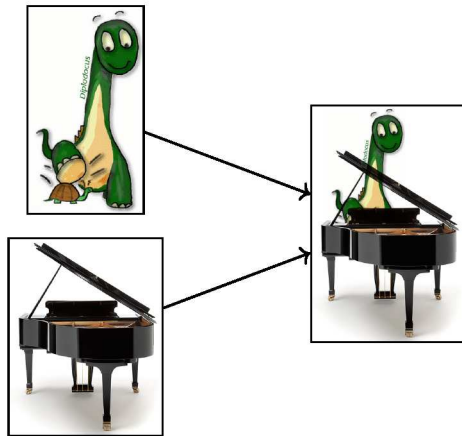
Methodology



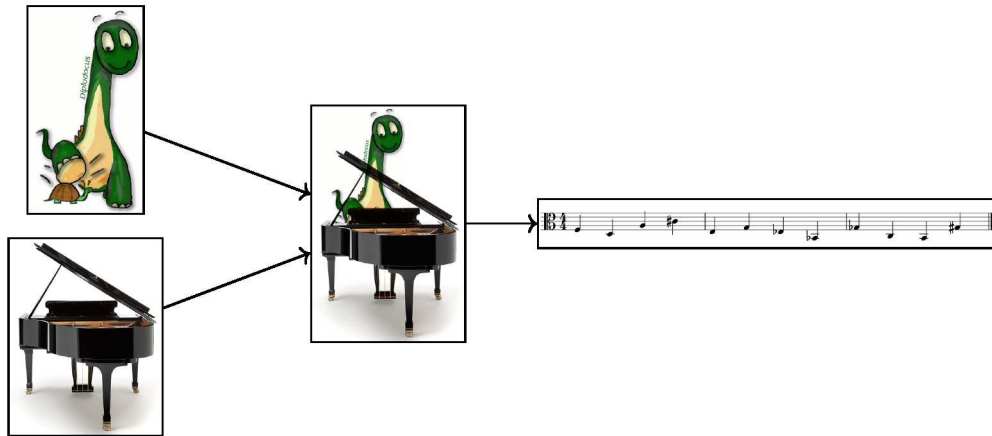
Piano Fazioli (www.pianoshanlet.fr)

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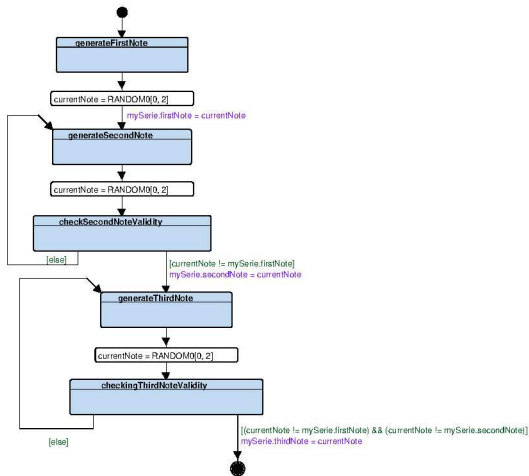
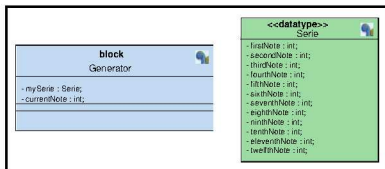
Methodology



Compose with a dinosaur Methodology



Compose with a dinosaur



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Essais Dodécaphoniques
Une œuvre sérielle générée par un dinosaure TTool, et un peu Bastien aussi

Al tempo di un diplodocus

The image shows a musical score for a piece titled 'Essais Dodécaphoniques'. The score is written for piano in 4/4 time. It consists of two systems of music. The first system has 7 measures, and the second system starts at measure 8 and has 5 measures. The music is a serial composition, featuring a sequence of notes and chords that change frequently. The tempo is marked 'Al tempo di un diplodocus'. The score is enclosed in a black rectangular frame.

Plan

Musical interpretation and fidelity to the score

The musical language: a formal language?

A set-theoretic approach

Serial music and *Pitch-Class Set Theory*

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And now?

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Research directions

Current approaches can be improved

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 $n = \langle h, d, t \rangle$

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- ▶ What about silence ?

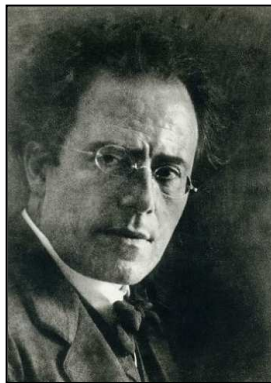
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- ▶ It could be interesting to consider jointly the three formal aspects of a note $n = \langle h, d, t \rangle$
- ▶ What about silence ?
- ▶ Evolution of musical languages

Modern music and modern mathematics

- ▶ 1908 : Zermelo set theory
- ▶ 1909-1910 : Mahler's 9th Symphony
- ▶ 1921 : Schoenberg's first serial work
- ▶ 1922 : ZF
- ▶ 1935 : Bourbaki
- ▶ 1949 : Darmstadt Summer Course, Messiaen creates *Mode de valeurs et d'intensité*



Gustav Mahler, photographed by Friedrich Victor Spitzer, 1905



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Bibliographie

Bibliographie I

- [1] Louis Allix.
L'authenticité comme norme de l'interprétation musicale.
Savoirs en prisme, (02):173–198, 2013.
- [2] Pierre Boulez.
Le concept d'écriture (I). Cours au Collège de France, 1990-1991, 1990.
- [3] Pierre Boulez et Alain Connes.
La créativité en musique et en mathématiques. Conférence IRCAM, 15 juin 2011.
https://medias.ircam.fr/x70ce3e_pierre-boulez-et-alain-connes-la-creativ, 06 2011.

Bibliographie II

- [4] Sviatoslav Richter et Bruno Monsaingeon.
Ecrits, conversations.
Arte editions. Van de Velde, 1998.
- [5] Allen Forte.
The structure of atonal music, volume 304.
Yale University Press, 1973.