Online optimization and machine learning: Applications to resource allocation problems in wireless networks

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E.V. Belmega, P. Mertikopoulos, and R. Negrel, "Online convex optimization in wireless networks and beyond: The feedback - performance trade-off", ***invited paper*** *at RAWNET intl. workshop in conjunction with WiOpt*, Sep. 2022.


I. Chafaa, R. Negrel, E.V. Belmega, and M. Debbah, "Self-supervised deep learning for mmWave beam steering exploiting sub-6 GHz channels", *IEEE Trans. on Wireless Commun.*, Mar. 2022.

I. Online resource optimization policies: feedback vs. performance

- Preliminaries
- First order (gradient) feedback
- Zeroth-order (scalar) feedback
- 1-bit feedback

II. Deep learning vs. online policies

- mmWave beamforming from sub-6GHz channels

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Distributed and dynamic wireless networks

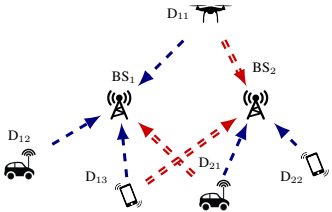
B5G, 6G, IoT [Saad'19, Tataria'21]

- **Characteristics and requirements:** dense, heterogeneous, autonomous, decentralized, energy-efficient, . . .
- **Challenges**

  Highly *mobile* radios & networks
  Unpredictable and *arbitrary* connectivity patterns
  Limited network/channel knowledge (*potentially outdated*)
  Limited processing capabilities

- Static or stochastic (optimal or Nash) solutions: not suitable
  $\Longrightarrow$ **dynamic, non-equilibrium** solutions

Obj: Design **energy-efficient** resource allocation policies coping with **arbitrary** network dynamics and **scarce** feedback



M transmitting devices, N receivers, S orthogonal bands

Shannon rate of an arbitrary device $R_t(\mathbf{p}(t)) = \frac{1}{S}\sum_{s=1}^{S} \log(1 + \overbrace{w_s(t)p_s(t)}^{SNR_s})$

$\mathbf{p}(t)$ - power allocation vector

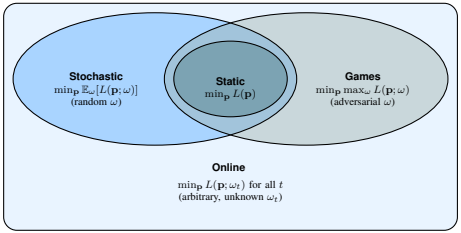
Effective channel gain $w_s(t) = \dfrac{|h_s(t)|^2}{\sigma^2 + \underbrace{\sum_j |h_s^j(t)|^2 p_s^j(t)}_{\text{interference}}}$

$$\text{minimize} \quad L_t(\mathbf{p}(t)) \triangleq \overbrace{\sum_{s=1}^{S} p_s(t)}^{\text{power consumption}} + \underbrace{\lambda\,[R_{\min} - R_t(\mathbf{p}(t))]^+}_{\text{minimum rate penalty}}$$

$$\text{s.t.} \quad p_s(t) \geq 0, \forall s, \ \sum_{s=1}^{S} p_s(t) \leq P_{\max}$$

Rk: **Arbitrary time-varying objective**, unknown at the transmission instant

- How to optimize an unknown $L(\mathbf{p}, \omega_t)$ objective at the decision time t?



Online iterative process

For $t = 1$ to T

- Choose policy $\mathbf{p}(t)$
- Incur loss $L(\mathbf{p}(t), \omega_t)$
- Receive **feedback**
- **Update**: $\mathbf{p}(t+1) \leftarrow \mathbf{p}(t)$ based on feedback

- **Goal:** develop *efficient* online processes based on strictly causal feedback
- Link with machine learning: **multi-armed bandits** (MABs) reinforcement learning
- **Assumptions:** on the objective function $L(\mathbf{p}, \omega_t)$ w.r.t. $\mathbf{p}$ but **not** on the underlying dynamics of ω_t

- Ideal benchmark

$$\forall t, \quad L_t(\mathbf{p}(t)) \quad - \quad \underbrace{\min_{\mathbf{q}\in\mathcal{P}} L_t(\mathbf{q})}_{\text{dynamic optimal loss}}$$

Too ambitious under the worse-case assumption of an arbitrarily time-varying network!

- Less ambitious benchmark: **regret** [Hannan'57]

$$\underbrace{Reg_T}_{\text{overall regret}} \triangleq \sum_{t=1}^{T} L_t(\mathbf{p}(t)) \quad - \quad \underbrace{\min_{\mathbf{q}\in\mathcal{P}} \sum_{t=1}^{T} L_t(\mathbf{q})}_{\text{optimal overall loss of } \textbf{fixed} \text{ policies}}$$

- **Rk:** both benchmarks require knowledge of the future!

Objective: design online policies $\mathbf{p}(t)$ that lead to **no regret**:

$$\limsup_{T\to\infty} \frac{1}{T} Reg_T \leq 0.$$

No-regret policies

Static convex optimization $L_t(\mathbf{p}) = L(\mathbf{p}), \forall t$

$\mathbf{p}(t)$ no-regret policy $\Rightarrow \overline{\mathbf{p}}(T) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{p}(t)$ converges to the optimal solution

Stochastic convex optimization $L_t(\mathbf{p}) = L(\mathbf{p}, \omega_t), \forall t$

$\mathbf{p}(t)$ no-regret policy $\Rightarrow \overline{\mathbf{p}}(T) = \frac{1}{T}\sum_{t=1}^{T}\mathbf{p}(t)$ converges to the optimal solution

Non-cooperative games [Viossat'13, Mertikopoulos'19]

A no-regret policy (cumulative) converges to the Nash equilibrium in
two-player (discrete or convex) **zero-sum** games, potential (discrete or convex) games, ...

Adversarial (non stochastic) MABs

- S arms of slot machines
- **Variable:** probability distribution $\mathbf{p}(t)$

$$\mathbf{p}(t) \in \mathcal{S} \triangleq \left\{ \mathbf{p} \in \mathbb{R}_+^S : \ \sum_{s=1}^S p_s = 1 \right\}$$

- **Objective:** maximize the linear expected gains $U_t(\mathbf{p}(t)) = \mathbf{r}(t)^T \mathbf{p}(t)$
  $r(t)$ - vector of arms' rewards at time t
- **Optimal policy**: exponential/multiplicative weights [Auer'02]

Our online power allocation problem

- S channels or frequency subcarriers
- **Variable:** power allocation vector $\mathbf{p}(t)$

$$\mathbf{p}(t) \in \mathcal{P} \triangleq \left\{ \mathbf{p} \in \mathbb{R}_+^S : \ \sum_{s=1}^S p_s \leq P_{\max} \right\}$$

- **Objective:** minimize the convex loss $L_t(\mathbf{p}(t))$ - tradeoff power consumption vs. rate
- **Hint:** adapted version of exponential weights

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Gradient-based online policies

Low complexity, parallel computing

Online Gradient Descent (OGD) **[Zinkevich'03]**

- Gradient feedback: $\mathbf{v}(t) = -\nabla L_t(\mathbf{p}(t))$
- Update: Euclidean projection of $\mathbf{p}(t) + \gamma \mathbf{v}(t)$

$$\begin{aligned} \mathbf{p}(t+1) &\triangleq \arg\min_{\mathbf{q}\in\mathcal{P}} \ \tfrac{1}{2}\|\mathbf{p}(t) + \gamma\mathbf{v}(t) - \mathbf{q}\|^2 \\ &= \arg\min_{\mathbf{q}\in\mathcal{P}} \ \left\{\gamma \mathbf{v}(t)^T(\mathbf{p}(t) - \mathbf{q}) + \tfrac{1}{2}\|\mathbf{p}(t) - \mathbf{q}\|^2\right\} \end{aligned}$$

Based on the mirror descent [Nemirovski'83]

Online Mirror Descent (OMD) **[Shalev-Shwartz'07]**

- Gradient feedback: $\mathbf{v}(t) = -\nabla L_t(\mathbf{p}(t))$
- Update: Mirror-mapping on $\mathbf{p}(t) + \gamma \mathbf{v}(t)$

$$\mathbf{p}(t+1) \triangleq \arg\min_{\mathbf{q}\in\mathcal{P}} \left\{\gamma \mathbf{v}(t)^T(\mathbf{p}(t) - \mathbf{q}) + D_h(\mathbf{q}, \mathbf{p}(t))\right\}$$

$D_h(\mathbf{q}, \mathbf{p}) \triangleq h(\mathbf{q}) - h(\mathbf{p}) - \nabla h(\mathbf{p})^T(\mathbf{q} - \mathbf{p})$ the Bregman divergence of regularizer $h(\cdot)$

Online Mirror Descent (OMD)

- Particular cases
  - OGD: Euclidean regularizer $h(\mathbf{p}) = \frac{1}{2}\|\mathbf{p}\|^2$, $D_h(\mathbf{q},\mathbf{p}) = \frac{1}{2}\|\mathbf{q}-\mathbf{p}\|^2$
  - Exponential weights for MAB:

    entropic regularizer $h(\mathbf{p}) = \sum_{s=1}^{S} p_s \log p_s$, Kullback-Leibler $D_h(\mathbf{q},\mathbf{p}) = \sum_{s=1}^{S} q_s \log \frac{q_s}{p_s}$
- MAB (simplex) regret performance: **scalability**
  - OGD: $Reg_T = \mathcal{O}\left(\sqrt{T\,S}\right)$ [Zinkevich'03]
  - Exponential weights: $Reg_T = \mathcal{O}\left(\sqrt{T\log S}\right)$ [Auer'02]

OMD allows to design algorithms that are **tailored to the feasible set** compared with OGD.

OXL based on gradient feedback

Main idea: we exploit the similarity of $\mathcal{P}$ with the simplex and the entropic regularizer

Online exponential learning (OXL)

- Gradient feedback: $\mathbf{v}(t) = -\nabla L_t(\mathbf{p}(t))$
- Update:

$$\mathbf{y}(t+1) = \mathbf{y}(t) + \gamma \mathbf{v}(t) \quad \text{cumulative gradient score}$$

$$p_s(t+1) = P_{\max} \frac{\exp(y_s(t+1))}{1+\sum_{i=1}^{S} \exp(y_i(t+1))}, \ \forall s \quad \text{exponential mapping}$$

- Similar to the exponential weights for MAB
- Closed-form update as opposed to OGD, which requires an Euclidean projection
- We proved: **no-regret** property, $Reg_T \leq P_{\max}\sqrt{2V \log(1+S)T}$, $V = \max \|v(t)\|^2$

  Extends to the imperfect gradient feedback case: $\tilde{\mathbf{v}}(t) = \mathbf{v}(t) +$ **error**

A. Marcastel, E. V. Belmega, P. Mertikopoulos, and I. Fijalkow, "Online power optimization in feedback-limited, dynamic and unpredictable IoT networks", IEEE Trans. on Signal Processing, 2019.

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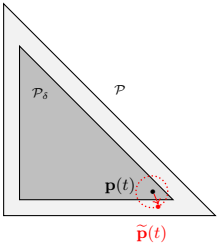
Can the feedback be reduced to a scalar?

- **Received feedback:** value of incurred loss $L_t(\mathbf{p})$ as in MABs with bandit feedback
- Stochastic gradient approximation: $\tilde{\mathbf{v}}(t) = \frac{S}{\delta} L_t(\mathbf{p} + \delta \mathbf{u})\, \mathbf{u}$, [Spall'97, Flaxman'05]
  $\mathbf{u}$ - uniformly drawn over the unit S-dimensional Sphere
- To estimate $\nabla L_t(\mathbf{p}(t))$, transmit at $\tilde{\mathbf{p}}(t) = \mathbf{p}(t) + \delta \mathbf{u} \longrightarrow \tilde{\mathbf{p}}(t)$ may fall outside $\mathcal{P}$!
- Our solution: **shrink the feasible set** s.t. $\forall\, \mathbf{p}(t) \in \mathcal{P}_\delta \subset \mathcal{P} \;\Rightarrow\; \tilde{\mathbf{p}}(t) \in \mathcal{P}$



$$\mathcal{P}_\delta \triangleq \left\{ \mathbf{p} \in \mathbb{R}_+^S : p_s \geq \delta, \sum_{s=1}^{S} p_s \leq P_{\max} - \sqrt{S}\delta \right\}$$

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Our modified OXL$_0$

OXL$_0$ algorithm

- Transmit at $\tilde{\mathbf{p}}(t) = \mathbf{p}(t) + \delta \mathbf{u}$
- Receive **scalar feedback:** $L_t(\tilde{\mathbf{p}}(t))$
- Gradient estimation: $\tilde{\mathbf{v}} = -\frac{S}{\delta} L_t(\tilde{\mathbf{p}}(t))\, \mathbf{u}$
- **Update:**

$$\mathbf{y}(t+1) = \mathbf{y}(t) + \gamma \tilde{\mathbf{v}}(t)$$ cumulative score

$$p_s(t+1) = \delta + P_{\max}(1 - C_\delta) \frac{\exp(y_s(t+1))}{1+\sum_{i=1}^{S} \exp(y_i(t+1))}, \ \forall s$$ modified exponential mapping adapted to the shrunk set $\mathcal{P}_\delta$

$C_\delta = \frac{\delta}{P_{\max}}(S + \sqrt{S})$

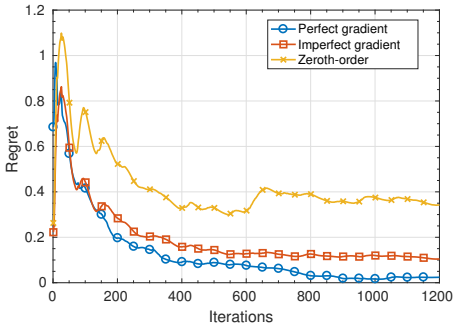
- We proved: **no-regret property**, $ERegT = \mathcal{O}(T^{3/4})$
- Zeroth-order feedback greatly impacts the decay rate of the average regret

Tradeoff: regret decay rate vs. amount of feedback!

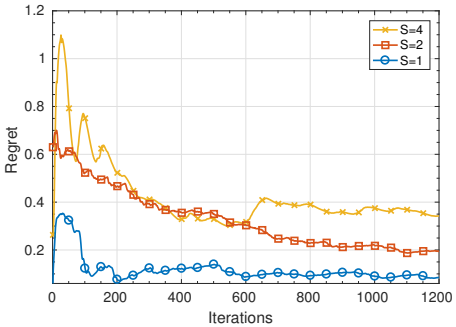
A. Marcastel, E. V. Belmega, P. Mertikopoulos, and I. Fijalkow, "Online power optimization in feedback-limited, dynamic and unpredictable IoT networks", IEEE Trans. on Signal Processing, 2019.

Impact of reducing the feedback

OXL vs. OXL_0



OXL_0 and S - problem dimension



Zeroth-order information reduces the decay rate of the average regret.
The decay rate becomes worse by increasing S - the problem dimensionality.

COST-HATA wireless channels, $M = 10$ transmitting devices, $S \in \{1, 2, 4\}$ bands, $N = 1$ receiver, $P_{\max} \in [0.5, 2]$, $R_{\min} \in [0.5, 3]$ bps/Hz, $\lambda \in [0.5, 10]$, average regret over 200 random draws of $\mathbf{u}$

Zeroth-order feedback with callbacks

- **Issues:** slow decay rate $\mathcal{O}(T^{3/4})$, poor scaling with problem dimensionality

  $\longrightarrow$ MXL0 performs poorly in MIMO networks

- Motivation: exponential weights yield $\mathcal{O}(\sqrt{T})$ regret in bandit feedback MABs

- Our idea: exploit the current feedback R_t jointly with the past R_{t-1}

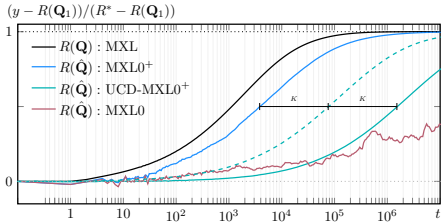
  $\longrightarrow$ improved two-point gradient estimator

- For **static convex objectives**, we can recover the $\mathcal{O}(\sqrt{T})$ regret as with perfect gradient feedback

K-user MIMO multiple access channel

$$R(\mathbf{Q}) = \log\det\left(\mathbf{I} + \sum\nolimits_{k=1}^{K} \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^{\dagger}\right)$$

$$\mathbf{Q}_k \succeq 0, \quad \mathrm{Tr}(\mathbf{Q}_k) \leq P_{\max}$$



Arbitrary static channels, $K = 20$ users, 3 transmit and 16 receive antennas

O. Bilenne, P. Mertikopoulos, and E.V. Belmega, "Fast Gradient-Free Optimization in Distributed Multi-User MIMO Systems", IEEE Trans. on Signal Processing, 2020.

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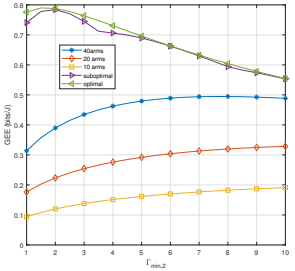
Can the feedback be reduced to one bit?

- **Our idea:** exploit MABs with outage (QoS) based rewards and ACK/NACK-type feedback
- Problems: beam alignment in mmWave networks, NOMA with no CSIT/CDIT
- **Issue:** feasible set needs to be quantized

  $\longrightarrow$ performance loss

2-user downlink NOMA

$$\mathrm{GEE}(i,\mathbf{p}) \triangleq \frac{(R_{\min,i}+R_{\min,j})\,(1-\mathbb{P}_{\mathrm{out}}(i,\mathbf{p}))}{p_i+p_j+P_c}$$



$\Gamma_{\min,2} = 2^{R_{\min,2}} - 1, \sigma_k^2 = 0.1, R_{\min,1} = 1$ bpcu, $P_c = 1$ W, $P_{\max} = 100$ W, $T = 5000, 10^3$ channel (Rayleigh) realizations

H. El Hassani, A. Savard, and E.V. Belmega, "Adaptive NOMA in time-varying wireless networks with no CSIT/CDIT relying on a 1-bit feedback", IEEE Wireless Commun. Lett., 2021.

- **Online optimization and no-regret learning:** a suitable framework for resource allocation problems in dynamic and unpredictable networks
- **Assumptions:** Convex (extends to fractional programs) and Lipschitz objectives $L(\mathbf{x}, \omega_t)$ w.r.t. $\mathbf{x}$, convex feasible sets

  $\Rightarrow$ **Adaptive online algorithms**

  Decentralized and reinforcing
  Cope with **arbitrary network dynamics**
  Rely on **strictly causal** information
  Imperfect and **reduced feedback**
  Regret-based theoretical **guarantees**

- Tradeoff: performance vs. available feedback
- Applies to semi-definite programming: in **massive MIMO** networks reducing the (matrix) gradient feedback is crucial

- Zeroth-order feedback with callbacks for **online** problems
- One-bit feedback policies: improve the performance, NOMA cope with continuous and discrete variables
- Beyond convex, Lipschitz objectives: more realistic (**non convex**) energy-efficiency measures
- Compare online algorithms with **deep learning** ones

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Model-based approaches tradeoff

simple (unrealistic) but tractable problems **vs.** *practical but not tractable problems*

$\Longrightarrow$ machine (deep) learning has the potential to bridge this gap

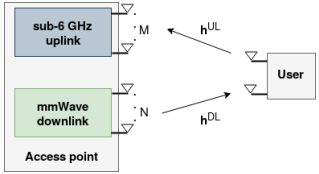
Deep learning based on neural networks [Zappone'19, Tataria'21]

- \+ Powerful toolbox: *universal approximators* of complex relationships
- \+ Pervasive in 6G network design, operation, ...
- − Large and relevant training datasets
- − High computing/processing capabilities

mmWave communications

- Exploit high frequency spectrum
- MIMO systems and **beam alignment** against severe pathloss
- **Challenges:** channel estimation, device mobility and network dynamics

  $\longrightarrow$ online optimization (MABs) and deep learning [Hashemi'18, Alrabeiah'20]
- **Idea:** Mapping sub-6GHz channels into mmWave beamforming vectors [Alrabeiah'20]

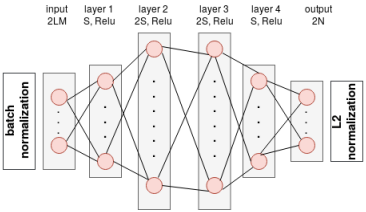


- Sub-6 GHz channels provide multipath signature, easier to estimate
- $\mathbf{h}^{\mathrm{UL}} \rightarrow \mathbf{f}^{\mathrm{DL}}$ highly complex and non linear mapping $\longrightarrow$ deep learning

Network architecture and DeepMIMO

Neural network architecture

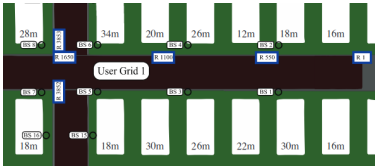
- Input: uplink sub-6 GHz channels
- Output: mmWave beamforming vectors
- **Fully-connected**: structure agnostic



Training

- DeepMIMO dataset: $\left(\{\mathbf{h}_i[\ell]^{\mathrm{UL}}\}_{\ell=1}^{L}, \{\mathbf{h}_i[\ell]^{\mathrm{DL}}\}_{\ell=1}^{L}\right)_i$
- Custom loss function: $\mathcal{L} = -1/\mathcal{B}\sum_{i=1}^{\mathcal{B}} \mathcal{R}_i$

$$\mathcal{R}_i = \frac{1}{L}\sum_{\ell=1}^{L} \log_2\left(1 + \frac{P^{\mathrm{DL}}}{L\,(\sigma^{\mathrm{DL}})^2} \mid \mathbf{h}_i^{\mathrm{DL}\dagger}[\ell]\,\mathbf{f}_i \mid^2\right)$$



DeepMIMO setup [Alkhateeb'19]

$\mathbf{f}_i$: predicted beamformer, P^{DL} transmit power, $\mathcal{B}$ size of mini-batch, L subcarriers, $(\sigma^{\mathrm{DL}})^2$ noise variance

I. Chafaa, R. Negrel, E.V. Belmega, and M. Debbah, "Unsupervised deep learning for mmWave beam steering exploiting sub-6 GHz channels", IEEE Trans. on Wireless Commun., 2022.

Deep learning or MABs?

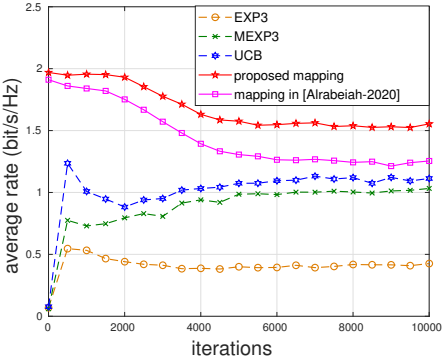
MABs

- \+ No *offline* training, online learning
- \+ Low complexity $\sim 10^2$ ops/iteration
- \+ 1-bit feedback (ACK/NACK)
- − Quantized beams $\longrightarrow$ performance loss
- − Trial and error $\longrightarrow$ transitory phase

Deep learning

- − Offline training on large and relevant datasets, *DeepMIMO* [Alkhateeb'19]
- − Higher complexity $\sim 10^6 - 10^7$ ops/iteration
- − sub-6 GHz channel information
- \+ Regression (no quantization)
- \+ Better running performance (no transitory phase)

Depends on the target application, its characteristics and requirements



MABs: EXP3, MEXP3, UCB

Deep learning [Alrabeiah'20]: classification

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I. Chafaa, E.V. Belmega, and M. Debbah, "One-bit Feedback exponential learning for beam alignment in mobile mmWave", IEEE Access, Oct. 2020.

- Deep learning approaches capture **complex relationships**:
  sub-6GHz uplink channels – downlink mmWave beams
- Extension to multi-cell multi-user networks
- **Robustness** to data imperfections
- **Generalization** capability: other wireless settings, problems
- Recurrent networks, reinforcement and online deep learning to capture the temporal network dynamics

AI-enhanced highly mobile and unpredictable IoT networks

OBJ-1 Design efficient **online optimization** algorithms
requiring **low-cost** and energy-efficient communication feedback

OBJ-2 Design online algorithms maximizing **non convex** energy efficiency
exploiting non convex online optimization and deep learning

Sustainable wireless communications: low-energy, low-cost and zero added electromagnetic waves

OBJ-1 Derive the fundamental **achievable Shannon rates**
in multi-user, multi-backscatter/RIS networks

OBJ-2 Develop **efficient algorithms** that jointly tune the transmit strategy
and the backscattering/RIS strategy

More intel on my webpage:
https://sites.google.com/site/evbelmega

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A. Marcastel, E.V. Belmega, P. Mertikopoulos, and I. Fijalkow, "Online power optimization in dynamic IoT networks: The impact of feedback scarcity", *IEEE Trans. on Signal Processing*, Mar. 2019.

P. Mertikopoulos, and E.V. Belmega, “Learning to be green: robust energy efficiency maximization in dynamic MIMO-OFDM systems”, *IEEE Journal on Selected Areas in Communication*, Apr. 2016.

H. El Hassani, A. Savard, and E.V. Belmega, “Energy-efficient 1-bit feedback NOMA in wireless networks with no CSIT/CDIT”, *IEEE SSP Workshop*, Jun. 2021.

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- Uplink signal: captured by AP at **sub-6 GHz**

$$\mathbf{y}^{\mathrm{UL}}[\ell] = \mathbf{h}^{\mathrm{UL}}[\ell]\, x^{\mathrm{UL}}[\ell] + \mathbf{n}^{\mathrm{UL}}[\ell]$$

$\mathbf{h}^{\mathrm{UL}}[\ell] \in \mathbb{C}^{M\times 1}$; $\mathbb{E}[\mid x^{\mathrm{UL}}[\ell] \mid^2] = P^{\mathrm{UL}}/L$; $\mathbf{n}^{\mathrm{UL}}[\ell] \sim \mathcal{N}(0, (\sigma^{\mathrm{UL}})^2)$

- Downlink signal: received by the user at the **mmWave band**

$$y^{\mathrm{DL}}[\ell] = \mathbf{h}^{\mathrm{DL}\,\dagger}[\ell]\, \mathbf{f}\, x^{\mathrm{DL}}[\ell] + n^{\mathrm{DL}}[\ell]$$

$\mathbf{h}^{\mathrm{DL}}[\ell] \in \mathbb{C}^{N\times 1}$; $\mathbf{f} \in \mathbb{C}^{N\times 1}, \|\mathbf{f}\|^2 = 1$; $\mathbb{E}[\mid x^{\mathrm{DL}}[\ell] \mid^2] = P^{\mathrm{DL}}/L$; $n^{\mathrm{DL}}[\ell] \sim \mathcal{N}(0, (\sigma^{\mathrm{DL}})^2)$

- Dataset samples: $(\{\mathbf{h}_i[\ell]^{\mathrm{UL}}\}_{\ell=1}^{L}, \{\mathbf{h}_i[\ell]^{\mathrm{DL}}\}_{\ell=1}^{L})_i$

  L number of OFDM subcarriers, i sample index

- Sub-6 GHz channels capture wireless characteristics, easy to estimate

- Dataset split at each link: 80% for training (15% validation set) and 20% for testing

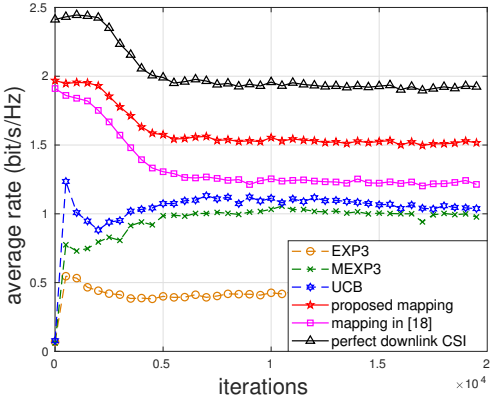
- System parameters:
  - Learning parameters: 100 epochs, ADAM optimizer (10^{-4}), $\mathcal{B} = 256$
  - Dataset parameters: 4 AP, outdoor scenario: 3.5 GHz $\rightarrow$ 28 GHz
  - Samples in each link: $108\,419$, $108\,781$, $63\,350$ and $117\,650$
  - $P^{\mathrm{DL}} = 34$ dBm, $M = 4$, $N = 64$ and $L = 32$, mmWave bandwidth 0.5 GHz
- Comparison benchmarks:
  - Centralized learning: same neural network, full access to all data samples
  - Perfect downlink CSI: full and instantaneous knowledge of downlink channels
  - Individual learning: fully distributed, no cooperation between APs

MABs

- \+ No *offline* training, online learning
- \+ Low complexity $\sim 10^2$ ops/iteration
- \+ 1-bit feedback (ACK/NACK)
- − Quantized beams $\rightarrow$ performance loss
- − Trial and error $\rightarrow$ transitory phase

Deep learning

- − Offline training on large and relevant datasets, *DeepMIMO* [Alkhateeb'19]
- − Higher complexity $\sim 10^6 - 10^7$ ops/iteration
- − sub-6 GHz channel information
- \+ Regression (no quantization)
- \+ Better running performance (no transitory phase)



MABs: EXP3, MEXP3, UCB

Deep learning [Alrabeiah'20]: classification

Depends on the target application, its characteristics and requirements

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MABs

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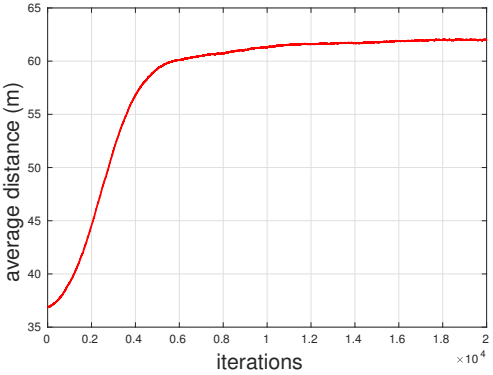

MABs: EXP3, MEXP3, UCB

Deep learning [Alrabeiah'20]: classification

Mobility model: average distance between the AP and user increases in time

Depends on the target application, its characteristics and requirements

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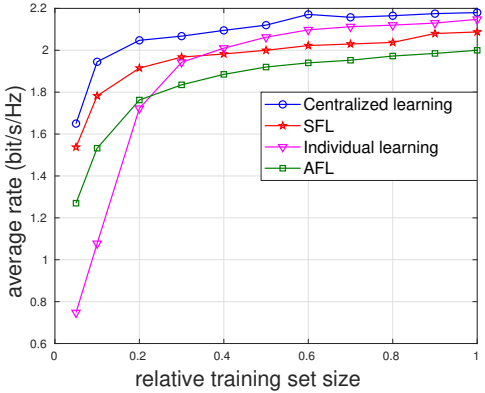
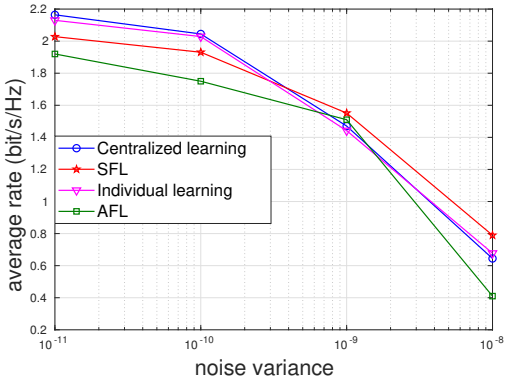
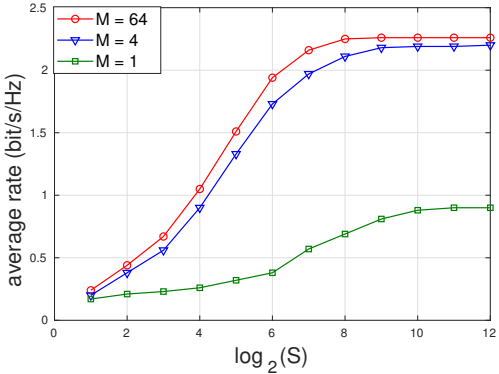


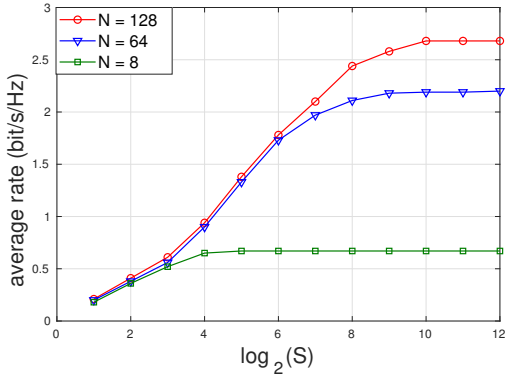
- FL approach is efficient in case of local data scarcity



- Uplink channels are contaminated with different levels of noise
- Variance of uplink channels: 10^{-9}
- SFL outperforms CL and individual learning at high noise regime



- $N = 64$ and $L = 32$
- When the input size M increases, the optimal size decreases



- $M = 4$ and $L = 32$.
- When the output size N increases, the optimal size increases as well

MABs

- UCB attains its ε-optimal performance (ε-regret) after $1/\varepsilon$ iterations (up to logarithmic factors)
- EXP3 and MEXP3 attains it after $1/\varepsilon^2$ iterations
- One iteration of these algorithms scales linearly with the codebook size: $\mathcal{O}(A)$

Deep learning

- Complexity of $\mathcal{O}(LMS + S^2 + SN)$ at each iteration

Entropic regularizer $h(\cdot)$: $D_h(\mathbf{y}, \mathbf{x}) = \sum_j y_j \log \frac{y_j}{x_j}$ (Kullback-Leibler divergence)

Exponential weights (EW / HEDGE / Multiplicative weights) [Auer'02]

- Full information (gradient feedback): $\mathbf{v}(t) \equiv \mathbf{r}(t) = \nabla U_t(\mathbf{p}(t))$ (reward vector)
- Recursive update (multiplicative):

$$p_s(t+1) = \frac{p_s(t)\exp(\gamma r_s(t))}{\sum_{i=1}^{S} p_i(t)\exp(\gamma r_i(t))}, \ \forall s$$

- Equivalent to:

$$\mathbf{y}(t+1) = \mathbf{y}(t) + \gamma \mathbf{r}(t) \quad \text{cumulative reward score}$$

$$p_s(t+1) = \frac{\exp(y_s(t+1))}{\sum_{i=1}^{S} \exp(y_i(t+1))}, \ \forall s \quad \text{exponential mapping}$$

- Regret bound: $Reg_T \leq \sqrt{2T \log S}$ with $\gamma^* = \sqrt{2 \log S/T}$

- Convex conjugate of the regularizer $h(\mathbf{p})$ - strongly convex:

$$\begin{aligned} h^*(\mathbf{y}) &= \max_{\mathbf{p}\in\mathcal{S}} \quad \mathbf{y}^T\mathbf{p} - h(\mathbf{p}) \\ &= \log\left(\sum_i \exp(y_i)\right) \end{aligned}$$

- Its gradient is equivalent with the update of EW algorithm $\mathbf{p}(t+1) = \nabla h^*(\mathbf{y}(t+1))$

$$\frac{\partial h^*(\mathbf{y})}{\partial y_i} = \frac{\exp(y_i)}{\sum_k \exp(y_k)}$$

- The update is also equivalent with

$$\mathbf{p}(t+1) = \arg\max_{\mathbf{p}\in\mathcal{S}} \quad \mathbf{y}(t)^T\mathbf{p} - h(\mathbf{p})$$

Feasible set: $\mathcal{P} = \left\{ \mathbf{p} \in \mathbb{R}_+^S : \sum_{s=1}^{S} p_s \leq P_{\max} \right\}$

Our regularizer $h(\mathbf{p}) = \sum_i p_i \log p_i + (P_{\max} - \sum_i p_i) \log(P_{\max} - \sum_i p_i)$

$D_h(\mathbf{q}, \mathbf{p}) = \sum_j q_j \log \frac{q_j}{p_j} + (P_{\max} - \sum_k q_k) \log \frac{P_{\max} - \sum_k q_k}{P_{\max} - \sum_k p_k}$

Kullback-Leibler type of divergence with additional variables:
$q_{s+1} = P_{\max} - \sum_{k=1}^{S} q_k, p_{s+1} = P_{\max} - \sum_{k=1}^{S} p_k$ (the unused power of zero reward)

OMD algorithm

- Gradient feedback: $\mathbf{v}(t) = -\nabla L_t(\mathbf{p}(t))$
- Update:

$$p_s(t+1) = \frac{P_{\max}\, p_s(t) \exp(\gamma v_s(t))}{\sum_{i=1}^{S} p_i(t) \exp(\gamma v_i(t)) + P_{\max} - \sum_k p_k(t)}, \quad \forall s$$

- Equivalent to OXL:

$$\mathbf{y}(t+1) = \mathbf{y}(t) + \gamma \mathbf{v}(t) \qquad \text{cumulative gradient score}$$

$$p_s(t+1) = P_{\max} \frac{\exp(y_s(t+1))}{1 + \sum_{i=1}^{S} \exp(y_i(t+1))}, \quad \forall s \qquad \text{exponential mapping}$$

- Convex conjugate of $h(\mathbf{p})$

$$\begin{aligned} h^*(\mathbf{y}) &= \max_{\mathbf{p}\in\mathcal{P}} \ \mathbf{y}^T\mathbf{p} - h(\mathbf{p}) \\ &= P_{\max} \ \log\left(1+\sum_i \exp(y_i)\right) - P_{\max}\log P_{\max} \end{aligned}$$

- $h(\mathbf{p})$ is $1/P_{\max}$ - strongly convex wrt $\|\cdot\|_1 \Longrightarrow h^*(\mathbf{y})$ is $P_{\max}$ strongly smooth wrt $\|\cdot\|_\infty$
- Its gradient is equivalent with the update of OXL algorithm: $\mathbf{p}(t+1) = \nabla h^*(\mathbf{y}(t+1))$

$$\frac{\partial h^*(\mathbf{y})}{\partial y_i} = P_{\max}\frac{\exp(y_i)}{1+\sum_k \exp(y_k)}$$

- The update is also equivalent with

$$\mathbf{p}(t+1) = \arg\max_{\mathbf{p}\in\mathcal{P}} \ \mathbf{y}(t)^T\mathbf{p} - h(\mathbf{p})$$

- First-order convexity condition of $L_t(\cdot)$ (linearization), $\mathbf{v}(t) = \nabla L_t(\mathbf{p}(t))$:

$$Reg_{\mathbf{q}}(T) \leq -\sum_{t=1}^{T} \langle \mathbf{v}(t) | \mathbf{p}(t) - \mathbf{q} \rangle$$

- Cumulative score: $\mathbf{y}(t+1) = \mathbf{y}(t) + \gamma \mathbf{v}(t)$ and $\mathbf{y}(1) = 0$

$$Reg_{\mathbf{q}}(T) \leq -\sum_{t=1}^{T} \langle \mathbf{v}(t) | \mathbf{p}(t) \rangle + \frac{1}{\gamma} \langle \mathbf{y}(T+1) | \mathbf{q} \rangle$$

- Using $\mathbf{p}(t) = \nabla h^*(\mathbf{y}(t))$ and $R_{\max}$-strong smoothness of $h^*(\mathbf{y})$

$$h^*(\mathbf{y}(t+1)) \leq h^*(\mathbf{y}(t)) + \gamma \langle \mathbf{v}(t) | \nabla h^*(\mathbf{y}(t)) \rangle + \frac{\gamma^2}{2} R_{\max} \|\mathbf{v}(t)\|_\infty^2,$$

$$Reg_{\mathbf{q}}(T) \leq \frac{1}{\gamma} \left[h^*(0) - h^*(\mathbf{y}(T+1)) \right] + \frac{\gamma}{2} R_{\max} \sum_{t=1}^{T} \|\mathbf{v}(t)\|_\infty^2 + \frac{1}{\gamma} \langle \mathbf{y}(T+1) | \mathbf{q} \rangle .$$

- Fenchel inequality: $h^*(\mathbf{y}(T+1)) + h(\mathbf{q}) \geq \langle \mathbf{y}(T+1) | \mathbf{q} \rangle$

$$Reg_{\mathbf{q}}(T) \leq \frac{1}{\gamma} \left[h(\mathbf{q}) + R_{\max} \log(1+S) - R_{\max} \log(R_{\max}) \right] + \frac{\gamma}{2} R_{\max} V T.$$

- From $h(\mathbf{q}) \leq R_{\max} \log(R_{\max})$

$$Reg(T) \leq \frac{R_{\max} \log(1+S)}{\gamma} + \frac{\gamma}{2} R_{\max} V T$$

- Fixed step-size γ

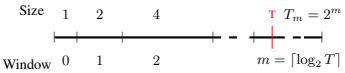
$$Reg_T \leq \overbrace{\frac{P_{\max}\log(1+S)}{\gamma}}^{\text{exploration penalty}} + \underbrace{\frac{\gamma\, P_{\max} V \mathbf{T}}{2}}_{\text{exploitation penalty}}$$

$V = \max \|\mathbf{v}(t)\|^2$

- If T **is known:** optimal step-size $\gamma^* = \sqrt{\frac{2\log(1+S)}{TV}}$

$$Reg_T \leq P_{\max}\sqrt{2V\log(1+S)T}$$

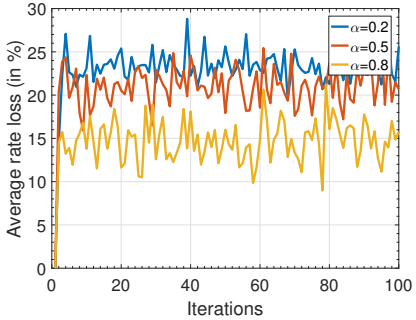
- If T **is not-known:** window doubling trick



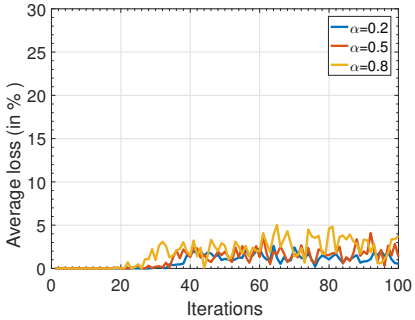
$$Reg_T \leq \frac{2}{\sqrt{2}-1} P_{\max}\sqrt{2V\log(1+S)T}$$

Average rate loss: $\frac{[R_{\min} - R(\mathbf{p})]^+}{R_{\min}}$

Waterfilling (energy-driven)



OXL



Having outdated feedback impacts the waterfilling performance at high channel variability.

Single device time-varying setup: $M = N = 1, S = 4, g(t+1) = \alpha g(t) + (1-\alpha) z(t)$ with $z(t) \sim \mathcal{N}(0, \sigma_z^2)$ (average over 90 draws), $R_{\max} = 2, R_{min} = 3$, $\lambda = 1$

- **Received feedback:** unbiased gradient estimation, $\tilde{\mathbf{v}}(t) = -\nabla L_t(\mathbf{p}(t)) + \text{error}$

$$\mathbb{E}\left[\tilde{\mathbf{v}}(t) \mid \mathcal{F}_{t-1}\right] = -\nabla L_t(\mathbf{p}(t)) \quad \text{no systematic errors}$$

$$\mathbb{E}\left[\|\tilde{\mathbf{v}}(t)\|^2 \mid \mathcal{F}_{t-1}\right] \leq \widetilde{V} \quad \text{bounded mean square}$$

$\mathcal{F}_t$ - history of play up to t

- OXL with $\mathbf{v}(t) \leftarrow \tilde{\mathbf{v}}(t)$ and an optimal step-size $\gamma^* = \sqrt{\frac{2\log(1+S)}{\widetilde{V}T}}$

$$\mathbb{E}\left[Reg_T\right] \leq P_{\max}\sqrt{2\widetilde{V}\ \log(1+S)\ T}$$

Rk: The expected regret is not highly impacted by unbiased errors in the gradient estimation.

- First-order convexity condition (linearization):

$$\mathbb{E}[Reg_{\mathbf{q}}(T)] \leq \mathbb{E}\left[\sum_{t=1}^{T} \langle \nabla L_t(\mathbf{p}(t)) | \mathbf{p}(t) - \mathbf{q} \rangle\right]$$

- Using $\nabla L_t(\mathbf{p}(t)) = -\mathbb{E}[\tilde{\mathbf{v}}(t)|\tilde{\mathbf{v}}(t-1), \ldots, \tilde{\mathbf{v}}(1)]$ and the law of total expectation

$$\mathbb{E}\left[\sum_{t=1}^{T} \langle \nabla L_t(\mathbf{p}(t)) | \mathbf{p}(t) - \mathbf{q} \rangle\right] = -\mathbb{E}\left[\sum_{t=1}^{T} \langle \tilde{\mathbf{v}}(t) | \mathbf{p}(t) - \mathbf{q} \rangle\right]$$

- Using $\mathbb{E}\left[\|\tilde{\mathbf{v}}(t)\|_2^2\right] \leq \tilde{V}$

$$\mathbb{E}[Reg(T)] \leq \frac{P_{\max} \log(1+S)}{\mu} + \frac{\mu}{2} P_{\max} T \tilde{V}$$

Stochastic gradient approximation: $\tilde{\mathbf{v}}(t) = \frac{S}{\delta}\, L_t(\mathbf{p} + \delta\mathbf{u})\, \mathbf{u}$, [Spall'97, Flaxman'05]

by sampling the loss function not at $\mathbf{p}$, but at a nearby point $\mathbf{u}$ - uniformly drawn over the unit S-dimensional Sphere

Properties:

$$\mathbb{E}\left[\tilde{\mathbf{v}}\right] = \nabla \tilde{L}_t(\mathbf{p}) \qquad \text{link with the gradient of } \tilde{L}_t(\mathbf{p})$$

$$\tilde{L}_t(\mathbf{p}) \triangleq \mathbb{E}\left[L_t(\mathbf{p} + \delta\mathbf{u})\right] \qquad \tilde{L}_t(\mathbf{p}) \text{ - smooth approximation of } L_t(\mathbf{p})$$

$$|L_t(\mathbf{p}) - \tilde{L}_t(\mathbf{p})| \leq L\delta$$

L - the Lipschitz constant of $L_t(\mathbf{p})$

- For fixed γ and δ

$$\mathbb{E}\left[Reg_T\right] \leq \frac{P_{\max}\ \log(1+S)}{2\gamma} + \gamma\ \mathbf{T}\ S^2\left(\frac{B}{\delta}+L\right)^2 + L\ \mathbf{T}\ \delta\ \left(3+P_{\max}\left(S+2\sqrt{S}\right)\right)$$

$B = \max L_t(\mathbf{p})$

- Choosing $\delta^* = \frac{P_{\max}}{(S+\sqrt{S})T^{1/4}}$ and $\gamma^* = \sqrt{\frac{P_{\max}\log(1+S)}{2T}}\left[S\left(\frac{B}{\delta^*}+L\right)\right]^{-1}$, the expected regret is sublinear

$$\mathbb{E}\left[Reg_T\right] = \mathcal{O}(T^{3/4}S^3\sqrt{\log(1+S)})$$

Rk: The zero-th order feedback greatly impacts the decay rate of the average regret.

$\rightarrow$ **Tradeoff:** regret decay rate vs. amount of required feedback!

New regularizer: $h(\mathbf{p}) = \sum_i (p_i - \delta)\log(p_i - \delta) + (P_{\max} - \sqrt{S}\delta - \sum_i p_i)\log(P_{\max} - \sqrt{S}\delta - \sum_i p_i)$

$D_h(\mathbf{q}, \mathbf{p}) = \sum_j (q_j - \delta)\log\frac{q_j - \delta}{p_j - \delta} + (P_{\max} - \sqrt{S}\delta - \sum_k q_k)\log\frac{P_{\max} - \sqrt{S}\delta - \sum_k q_k}{P_{\max} - \sqrt{S}\delta - \sum_k p_k}$

Kullback-Leibler type of divergence adapted to the shrunk set

OMD algorithm

- Gradient feedback: $\mathbf{v}(t) = -\nabla L_t(\mathbf{p}(t))$
- Update:

$$p_s(t+1) = \delta + \frac{P_{\max}(1 - C_\delta)\,(p_s(t) - \delta)\exp(\gamma v_s(t))}{\sum_{i=1}^{S}(p_i(t) - \delta)\exp(\gamma v_i(t)) + P_{\max} - \sqrt{S}\delta - \sum_k p_k(t)}, \quad \forall s$$

- Equivalent to OXL_0:

$$\mathbf{y}(t+1) = \mathbf{y}(t) + \gamma\tilde{\mathbf{v}}(t) \qquad \text{cumulative score}$$

$$p_s(t+1) = \delta + P_{\max}(1 - C_\delta)\frac{\exp(y_s(t+1))}{1 + \sum_{i=1}^{S}\exp(y_i(t+1))}, \quad \forall s \qquad \text{modified exponential mapping adapted to the shrunk set } \mathcal{P}_\delta$$

$C_\delta = \frac{\delta}{P_{\max}}(S + \sqrt{S})$

- Convex conjugate of $h(\mathbf{p})$:

$$\begin{aligned} h^*(\mathbf{y}) &= \max_{\mathbf{p}\in\mathcal{P}_\delta} \ \mathbf{y}^T\mathbf{p} - h(\mathbf{p}) \\ &= \delta \sum_i y_i + P_{\max}(1-C_\delta)\ \log\left(1+\sum_i \exp(y_i)\right) \\ &\quad -P_{\max}(1-C_\delta)\log(P_{\max}(1-C_\delta)) \end{aligned}$$

- Its gradient is equivalent with the update of OXL algorithm: $\mathbf{p}(t+1) = \nabla h^*(\mathbf{y}(t))$

$$\frac{\partial h^*(\mathbf{y})}{\partial y_i} = \delta + P_{\max}(1-C_\delta)\ \frac{\exp(y_i)}{1+\sum_k \exp(y_k)}$$

- The update is also equivalent with

$$\mathbf{p}(t+1) = \arg\max_{\mathbf{p}\in\mathcal{P}_\delta} \ \mathbf{y}(t)^T\mathbf{p} - h(\mathbf{p})$$

- Compare $L_t(\mathbf{p}_\delta(t)+\delta\mathbf{u})$, $L_t(\mathbf{q})$ to $L_t(\mathbf{p}_\delta(t))$, $L_t(\mathbf{q}_\delta)$ using the L-Lipschitz property (move to the shrunk set)

$$\mathbb{E}[Reg_{\mathbf{q}}(T)] \leq \mathbb{E}\left[\sum_{t=1}^{T} L_t(\mathbf{p}_\delta(t)) - L_t(\mathbf{q}_\delta)\right] + LT\delta\left(1+P_{\max}\left(S+2\sqrt{S}\right)\right)$$

- Compare $L_t(\mathbf{p}_\delta(t))$, $L_t(\mathbf{q}_\delta)$ to smooth approx. $\tilde{L}_t(\mathbf{p}_\delta(t))$, $\tilde{L}_t(\mathbf{q}_\delta)$ for linking the regret with $\tilde{v}(t)$

$$\mathbb{E}[Reg_{\mathbf{q}}(T)] \leq \mathbb{E}\left[\sum_{t=1}^{T} \tilde{L}_t(\mathbf{p}_\delta(t)) - \tilde{L}_t(\mathbf{q}_\delta)\right] + LT\delta\left(3+P_{\max}\left(S+2\sqrt{S}\right)\right)$$

- First-order convexity condition of $\tilde{L}_t(\mathbf{p})$: $\mathbb{E}\left[\sum_{t=1}^{T} \tilde{L}_t(\mathbf{p}_\delta(t)) - \tilde{L}_t(\mathbf{q}_\delta)\right] \leq \mathbb{E}\left[\sum_{t=1}^{T} \langle \nabla \tilde{L}_t(\mathbf{p}_\delta(t))|\mathbf{p}_\delta(t)-\mathbf{q}_\delta\rangle\right]$
- Using $\nabla \tilde{L}_t(\mathbf{p}_\delta(t)) = \mathbb{E}[\tilde{\mathbf{v}}(t)|\mathbf{u}(1),\ldots,\mathbf{u}(t-1)]$ and the total law of expectation:
  $\mathbb{E}\left[\sum_{t=1}^{T} \langle \nabla \tilde{L}_t(\mathbf{p}_\delta(t))|\mathbf{p}_\delta(t)-\mathbf{q}_\delta\rangle\right] \leq -\mathbb{E}\left[\sum_{t=1}^{T} \langle \tilde{\mathbf{v}}(t)|\mathbf{p}_\delta(t)-\mathbf{q}_\delta\rangle\right]$
- Using the convex conjugate of $h(\mathbf{p}_\delta)$, the links with the update and $||\tilde{v}(t)||_\infty^2 \leq S^2(B/\delta+L)^2$

$$\mathbb{E}[Reg(T)] \leq \frac{P_{\max}\log(1+S)}{2\gamma} + \gamma T S^2\left(\frac{B}{\delta}+L\right)^2 + LT\delta\left(3+P_{\max}\left(S+2\sqrt{S}\right)\right).$$

where $B = \max_{t,\mathbf{p}} L_t(\mathbf{p})$

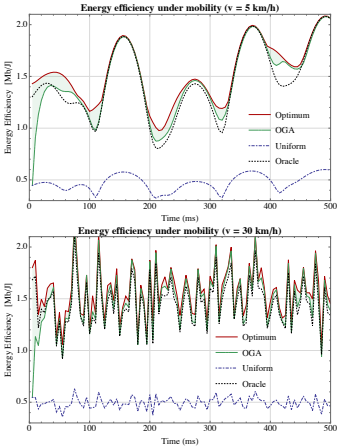
- Based on **gradient feedback**, we proposed

  matrix gradient descent and exponential learning for rate/energy-efficiency maximization

- Exponential mapping preserves the positivity of the input covariance matrices

- Our online policies perform close to the **ideal benchmark:** dynamic optimal one

COST-HATA wireless channels, mobile users $[3 - 130]$ km/h, $K = 15$ users, 4×8 MIMO, $S = 8$ bands



P. Mertikopoulos, and E.V. Belmega, "Learning to be green: robust energy efficiency maximization in dynamic MIMO-OFDM systems", IEEE JSAC, vol. 34, no. 4, pp. 743 – 757, Apr. 2016.

- Multi-antenna devices: M_k transmit antennas at user $k \in \{1, \ldots, K\}$, N receive antennas
- Achieved Shannon sum rate

$$R(\mathbf{Q}) = \log\det\left(\mathbf{I} + \sum_{k=1}^{K} \mathbf{H}_k \mathbf{Q}_k \mathbf{H}_k^{\dagger}\right)$$

$\mathbf{H}_k \in \mathbb{C}^{N \times M_k}$ - channel matrix between transmitter k and receiver, $\mathbf{Q} = (\mathbf{Q}_1, \ldots, \mathbf{Q}_K)$, $M = \max_k M_k$

- Distributed optimization: each transmitter k tunes its input covariance matrix $\mathbf{Q}_k \in \mathbb{C}^{M_k \times M_k}$ s.t.

$$\mathbf{Q}_k \succeq 0, \quad \mathrm{Tr}(\mathbf{Q}_k) = P_{\max,k}$$

*P. Mertikopoulos, and **E.V. Belmega**, "Learning to be green: robust energy efficiency maximization in dynamic MIMO-OFDM systems", IEEE JSAC, vol. 34, no. 4, pp. 743 – 757, Apr. 2016.*

- **Assumptions**: static channels, $S = 1$
- Rate maximization algorithms based on **gradient feedback**:

  MXL - matrix exponential learning [MertikopoulosB'16]
  IWF - iterative waterfilling [Yu'04]

$$\nabla_k R(\mathbf{Q}) = \mathbf{H}_k^\dagger \left[\mathbf{I} + \sum_{\ell=1}^{K} \mathbf{H}_\ell \mathbf{Q}_\ell \mathbf{H}_\ell^\dagger \right]^{-1} \mathbf{H}_k$$

- Massive MIMO systems: $N >> M, K >> 1$

  Issue: too much feedback $\mathcal{O}(\min\{KM^2, N^2\})$ at each iteration!

MXL algorithm **[MertikopoulosB'16]**

- Transmit at $\mathbf{Q}(t)$
- Receive **matrix** gradient feedback: $\mathbf{V}(t) = \nabla_k R(\mathbf{Q}(t))$
- **Update:**

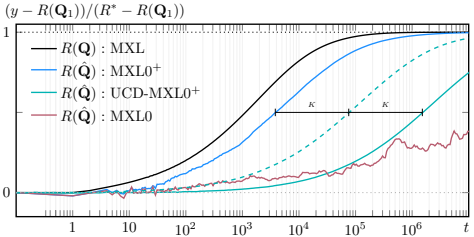
$$\mathbf{Y}(t+1) = \mathbf{Y}(t) + \gamma \mathbf{V}(t)$$

$$\mathbf{Q}(t+1) = \Lambda(\mathbf{Y}_t), \ \Lambda_k = P_{\max,k} \frac{\exp(\mathbf{Y}_k)}{\mathrm{tr}(\exp(\mathbf{Y}_k))}$$

- MXL with **utility-based gradient estimator:** feedback $R(t) = R(\mathbf{Q}(t))$, slow convergence $\mathcal{O}(T^{-1/4})$, poor scaling with problem dimensionality (MXL0)

- **Idea:** exploit the current sum rate feedback $R(t)$ jointly with the previous one $R(t-1) = R(\mathbf{Q}(t-1))$ (MXL0$^+$)

  $\longrightarrow$ improved two-point gradient estimator with no additional feedback



Arbitrary static channels, $K = 20$ users, 3 transmit and 16 receive antennas

Remark: for **static** convex and Lipschitz objectives, MXL0$^+$ recovers the convergence rate $\mathcal{O}(1/\sqrt{T})$ as MXL with perfect gradient feedback !

O. Bilenne, P. Mertikopoulos, and E.V. Belmega, "Fast Gradient-Free Optimization in Distributed Multi-User MIMO Systems", IEEE Trans. on Signal Processing, Oct. 2020.

- First-order derivative: $\ell'(x) = \lim_{\delta \to 0} \frac{\ell(x+\delta) - \ell(x-\delta)}{2\delta}$
- A two query estimator for small δ: $\hat{v}_2 = \frac{\ell(x+\delta) - \ell(x-\delta)}{2\delta}$
- One query random estimator: $u \sim \text{Bernoulli}(1/2)$

$$\hat{v} = \frac{\ell(x + \delta u)u}{\delta}, \qquad \mathbb{E}[\hat{v}] = \hat{v}_2$$

Bias: $|\mathbb{E}[\hat{v}] - v| = \mathcal{O}(\delta)$, variance: $|\hat{v}| = \mathcal{O}(1/\delta)$

- One query with callback ρ:

$$\hat{v}_\rho = \frac{(\ell(x + \delta u) - \rho)\, u}{\delta}, \qquad \rho = \ell(x)$$

Bias: $|\mathbb{E}[\hat{v}_\rho] - v| = \mathcal{O}(\delta)$, variance: $|\hat{v}_\rho| \leq L$
L - Lipschitz constant of ℓ

- Ensure the query point falls in the feasible set

$$\hat{\mathbf{Q}}_k = \mathbf{Q}_k + \frac{\delta}{r_k}(\mathbf{C}_k - \mathbf{Q}_k) + \delta \mathbf{Z}_k \quad \in \mathcal{Q}_k$$

K users, M_k transmit antennas, N receive antennas, $M = \max_k M_k$

$\mathbf{C}_k = \mathbf{I}/M_k, r_k = 1/\sqrt{M_k(M_k-1)}, \delta < r_k$

$\mathbf{Z}_k$ uniformly drawn on the unit sphere, $d_k = M_k^2 - 1$ dimension of $\mathcal{Q}_k$

- One-point estimator: $\hat{\mathbf{V}}_k(\mathbf{Q}) = \frac{d_k}{\delta} R(\hat{\mathbf{Q}})\mathbf{Z}_k$
- One-point estimator with callback: $\tilde{\mathbf{V}}_{k,t}(\mathbf{Q}) = \frac{d_k}{\delta}\left[R(\hat{\mathbf{Q}}_t) - R(\hat{\mathbf{Q}}_{t-1})\right]\mathbf{Z}_{k,t}$
- **Remark:** our algorithm does not require two queries at each iteration t!
- Convergence: $\mathcal{O}(\sqrt{K^4 M^6 \log M/T})$ with $\gamma \propto 1/\sqrt{T}, \delta \propto 1/\sqrt{T}$

Qu: Can we design an *adaptive NOMA* scheme via MABs relying on **little feedback** and **no CSIT/CDIT**?

Main challenge: Unknown channels that vary in time

- Simple case: $K = 2$ (or user pairing)
- Decoding order is a *discrete* control variable $i \in \{1, 2\}$
  - User $i \in \{1, 2\}$ performs SIC
  - User $j \neq i$ suffers interference
- Fix the transmit rates $R_{\min,k}$ and measure the outage

$$\mathbb{P}_{\text{out}}(i, \mathbf{p}) = \mathbb{P}[R_i(t) \leq R_{\min,i} \cup \min\{R_{j\to j}(t), R_{j\to i}(t)\} \leq R_{\min,j}]$$

- Feedback: 1-ACK/NACK bit from each user [ElHassaniSB'21a, ChafaaBD'20]

$$R_i(t) = \log\left(1 + \frac{|h_i(t)|^2 p_i}{\sigma_i^2}\right), \; R_{j\to j}(t) = \log\left(1 + \frac{|h_j(t)|^2 p_j}{|h_j(t)|^2 p_i + \sigma_j^2}\right), \; R_{j\to i}(t) = \log\left(1 + \frac{|h_i^{(t)}|^2 p_j}{|h_i^{(t)}|^2 p_i + \sigma_i^2}\right)$$

H. El Hassani, A. Savard, and E. V. Belmega, "Adaptive NOMA in time-varying wireless networks with no CSIT/CDIT relying on a 1-bit feedback", accepted paper IEEE Wireless Commun. Lett., Nov. 2020.

- Ideal energy-efficient target [Zhang'20]

$$\begin{aligned} \text{maximize} \quad & \mathrm{GEE}(i,\mathbf{p}) \triangleq \frac{(R_{\min,i}+R_{\min,j})\,(1-\mathbb{P}_{\text{out}}(i,\mathbf{p}))}{p_i+p_j+P_c} \\ \text{subject to} \quad & i\in\{1,2\}, \quad \mathbf{p}=(p_i,p_j)\in\mathbb{R}_+^2, \;\; p_i+p_j\le P_{\max} \end{aligned}$$

- Quantization of the power allocation
  - User fairness: less power is allocated to the SIC decoding user i

    $\mathbf{p}_\beta = (0.25\,\beta\,P_{\max}, 0.75\,\beta\,P_{\max})$, for discrete $\beta\in\mathcal{B}$
  - Uniformly quantized $\mathcal{B}\subset[0,1]$
- An arm $\mathbf{a}\triangleq(i,\mathbf{p}_\beta)\in\mathcal{A}$ decoding and power allocation vector

$$\mathcal{A}=\{1,2\}\times\{\mathbf{p}_\beta=(0.25\,\beta\,P_{\max},0.75\,\beta\,P_{\max})\mid\beta\in\mathcal{B}\}$$

Rk: The quantization and the $1/4-3/4$ user split will imply an optimality loss.

Adaptive energy-efficient 1-bit feedback NOMA [ElHassaniSB'21b]

Initialize: $t = 1, \mathbf{a}(1) = (1, 0.5)$

For $t = 1$ to T

- Play arm $\mathbf{a}(t) = (i(t), \beta(t))$
  - Inform users of their decoding: $i(t)$
  - Transmit with $\mathbf{p}_{\beta}(t) = (0.25\,\beta(t)\,P_{\max},\ 0.75\,\beta(t)\,P_{\max})$
- Receive 1-bit ACK/NACK feedback from each user
- Compute reward

$$U_t(\mathbf{a}(t)) = \begin{cases} \frac{R_{\min,1}+R_{\min,2}}{\beta(t)P_{\max}+P_c}, & \text{if } R_i(t,\mathbf{a}(t)) \geq R_{\min,i} \text{ and} \\ & \min\{R_{j\to j}(t,\mathbf{a}(t)),\, R_{j\to i}(t,\mathbf{a}(t))\} \geq R_{\min,j} \\ 0, & \text{otherwise} \end{cases}$$

- Update policy $\mathbf{a}(t+1) \leftarrow \mathbf{a}(t)$ via UCB or EXP3
- $t \leftarrow t + 1$

- Both UCB and EXP3 have the no regret property

  $\rightarrow$ adaptive NOMA reaches the best expected reward

$$\max_{\mathbf{a}\in\mathcal{A}} \mathbb{E}[U_t(\mathbf{a})] \equiv \max_{i\in\{1,2\},\beta\in\mathcal{B}} GEE(i,\mathbf{p}_\beta)$$

- **Regret** decay rate: $Reg_T^{\mathrm{UCB}} = \mathcal{O}(\log T/T)$ and $Reg_T^{\mathrm{EXP3}} = \mathcal{O}(1/\sqrt{T})$

- **Complexity** of UCB and EXP3: every iteration scales as $\mathcal{O}(|\mathcal{A}|)$

Rk: There are two fundamental tradeoffs: performance vs. feedback information and performance vs. complexity.

H. El Hassani, A. Savard, and E. V. Belmega, "Adaptive NOMA in time-varying wireless networks with no CSIT/CDIT relying on a 1-bit feedback", submitted paper IEEE SSP, Feb. 2021.

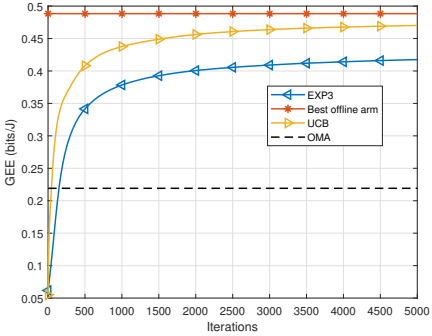
- Benchmark quantized OMA

$$R_k^{\mathrm{OMA}}(t) = \tfrac{1}{2}\log\left(1+\frac{|h_k(t)|^2\beta P_{\max}}{\sigma_k^2}\right)$$

$$GEE^{\mathrm{OMA}}(\beta) = \frac{(R_{\min,1}+R_{\min,2})(1-\mathbb{P}_{\mathrm{out}}^{\mathrm{OMA}}(\beta))}{\beta P_{max}+P_c}$$

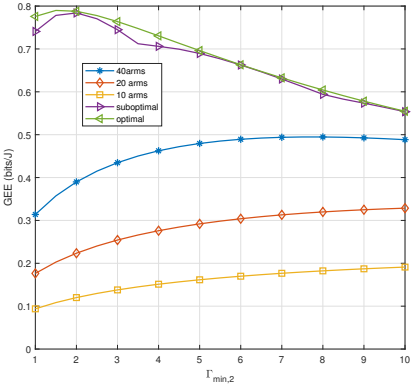
- Assumes perfect CDIT

$\max_{\beta\in\mathcal{B}} GEE^{\mathrm{OMA}}(\beta)$

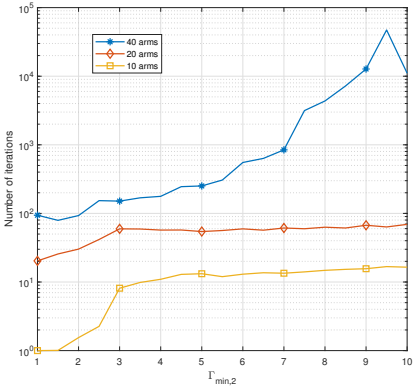


Our 1-bit feedback adaptive NOMA scheme outperforms quantized OMA with perfect CDIT.

$\sigma_k^2 = 0.1$, $R_{\min,1} = 1$ bpcu, $R_{\min,2} = 10$ bpcu, $P_c = 1$ W, $P_{\max} = 100$ W, $T = 5000, 10^3$ channel (Rayleigh) realizations



Energy efficiency as a function of $\Gamma_{\min,2} = 2^{R_{\min,2}} - 1$



Number of iterations to reach a 10% regret level

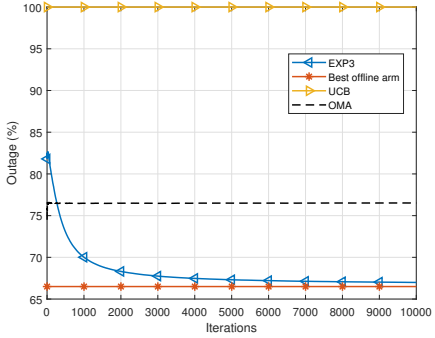
f

Our adaptive 1-bit feedback NOMA scheme is sub-optimal compared to the ideal target (with CDIT).

Outage minimization problem

Malicious jammer

- Has access to perfect CSI
- Anticipates the arm that will be played via **deterministic** UCB
- Interferes such that the system is put systematically in outage



In non-stationary adversarial settings EXP3 reaches no regret
while UCB is brought to a fault.

H. El Hassani, A. Savard, and E. V. Belmega, "Adaptive NOMA in time-varying wireless networks with no CSIT/CDIT relying on a 1-bit feedback", accepted paper IEEE Wireless Commun. Lett., Nov. 2020.

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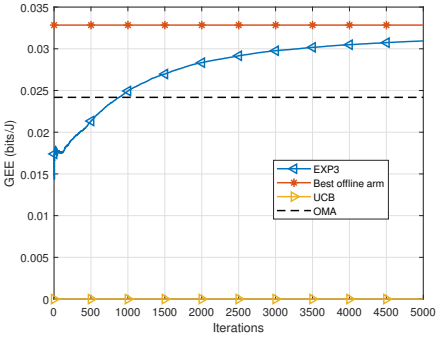

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H. El Hassani, A. Savard, and E. V. Belmega, "Adaptive NOMA in time-varying wireless networks with no CSIT/CDIT relying on a 1-bit feedback", accepted paper IEEE Wireless Commun. Lett., Nov. 2020.

- They **tradeoff between data exploration and exploitation**

  $\rightarrow$ poorly chosen lead to **positive regret**

- From a theoretical perspective (to reach no regret asimptotically)

$$\eta^* > 2 \quad \text{and} \quad \gamma^* = \sqrt{\frac{|\mathcal{A}| \ln |\mathcal{A}|}{(e-1)T}}$$

- But these values minimize the upper-bound of the regret

  $\rightarrow$ not optimal for the actual regret

- They can be further shaped via empirical experiments

Online mirror descent for convex optimization

- **Regret guarantees** $\mathcal{O}(\sqrt{T})$ rely on Lipschitz continuity

$$|\ell(\mathbf{x}) - \ell(\mathbf{y})| \leq G\|\mathbf{x} - \mathbf{y}\| \quad \text{or} \quad \|\text{grad}\, \ell(\mathbf{x})\| \leq G$$

- We extend them to problems with **singular gradients** of the objective at the boundary of the feasible set
- **Idea:** generalize the Lipschitz continuity based on local Riemannian norms, $\|\mathbf{z}\|_x = \mathbf{z}^T \mathbf{G}(\mathbf{x})\mathbf{z}$ with $\mathbf{G}(\mathbf{x}) \succeq 0$

Collaborator: P. Mertikopoulos (CNRS)
Student: K. Antonakopoulos (PhD)
Supported by: Inria, ANR ORACLESS
Publications: 3 confs.

e.g. $\ell(\mathbf{x}) = -\log(\mathbf{a}^T\mathbf{x})$, $\|\text{grad}\, \ell\|^2 = \frac{\sum_i a_i^2}{(\mathbf{a}^T\mathbf{x})^2}$

$\mathbf{G}(\mathbf{x}) = \mathbf{I} / \left(\sum_i x_i\right)^2 \Rightarrow \|\text{grad}\, \ell\|_x^2 \leq \frac{\sum_i a_i^2}{(\min_j a_j)^2}$

*K. Antonakopoulos, E.V. Belmega, and P. Mertikopoulos, "Online and stochastic optimization beyond Lipschitz continuity: A Riemannian approach", ICLR, **spotlight talk**, 2020.*

- Applications: tomography, astronomical data
- Ill-conditioned reconstruction problems: $\hat{\mathbf{u}} = \mathbf{H}\mathbf{x} + \hat{\mathbf{z}} \quad \dim \hat{\mathbf{u}} \ll \dim \mathbf{x}$

  $\mathbf{H}$ - (Toeplitz, circulant, convoluting) optical system, $\hat{\mathbf{z}}$ - Poisson noise

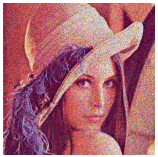
  $\hat{u}_i \sim \mathrm{Poisson}(\mathbf{H}\mathbf{x})_i$

- **Objective:** minimize Kullback-Leibler divergence

  $\ell(\mathbf{x}, \hat{\mathbf{u}}) = \sum_i [\hat{u}_i \log(\hat{u}_i/(\mathbf{H}\mathbf{x})_i) + (\mathbf{H}\mathbf{x})_i - \hat{u}_i]$

- Example: test image contaminated with Poisson noise

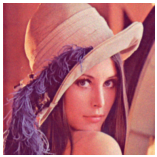
Poisson data



LR reconstruction

CMP reconstruction

RMD reconstruction



$\dim \hat{u} = \dim \hat{x}/2, \dim \hat{x} \simeq 10^5$

LR - Lucy-Richardson, mirror descent with entropic regularizer, [Bertero-2009]

CMD - mirror descent with log-barrier regularizer, [He-2016]

RMD - mirror descent with local Riemann norm

*K. Antonakopoulos, E.V. Belmega, and P. Mertikopoulos, "Online and stochastic optimization beyond Lipschitz continuity: A Riemannian approach", ICLR, **spotlight talk**, 2020.*

- Data **similarity** search and classification
  Mahalanobis distance:

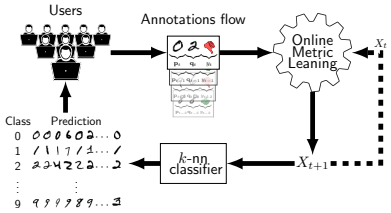
$$d_X(\mathbf{p}, \mathbf{q}) = \|\mathbf{X}^{1/2}\mathbf{p} - \mathbf{X}^{1/2}\mathbf{q}\|^2$$

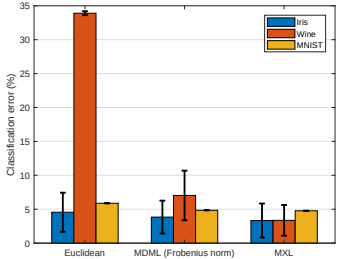
$\mathbf{p}$, $\mathbf{q}$ - input data, e.g., images

- **Objective:** learn the best discriminative linear data transformation matrix $\mathbf{X}$

$$\mathbf{X} \succeq 0, \quad \mathrm{Tr}(\mathbf{X}) \leq c$$

- Training examples are obtained online, **on-the-fly**
- Our matrix exponential learning is competitive in a toy setup
- Other examples: online matrix completion, universal linear filtering, online dictionary learning





MDML - mirror descent with Frobenius norm regularizer [Kunapuli-2012]

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